The Dynamics of Development: Entrepreneurship, Innovation, and Reallocation

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Abstract

Development dynamics are characterized by sustained improvements in TFP, protracted increases in investment rates, and a broad transformation in the structure of production. Low income countries are characterized by small average firm size, slow firm growth over the life-cycle, and significant dispersion of marginal products. In this paper we present a quantitative theory that jointly matches the behavior of firms in under-developed economies and key properties of development paths. We work with a model that features endogenous innovation decisions by entrepreneurs, reallocation of factors due to idiosyncratic productivity shocks, and selection in and out of entrepreneurship. We construct a low-TFP stationary equilibrium with dispersion in marginal products that is driven by idiosyncratic distortions. We then trigger development through a reform that liberalizes the economy from all frictions. Our quantitative theory can account well for cross-sectional and life-cycle patterns in distorted economies, and can generate development paths with rising TFP and investment dynamics, consistent with the data. Ignoring either endogenous innovation or selection in and out of entrepreneurship would lead to counter-factual transition paths, similar to those of the standard neoclassical growth model.

1 Introduction

There are significant divergences in firm behavior between developed and underdeveloped countries. Firms are on average smaller in the latter, grow only slowly over the life-cycle, and operate at scales that are typically inadequate for their levels of productivity. A burgeoning literature emerged trying to understand the implications of these facts for explaining cross-country differences in income and productivity. However less attention has been paid to investigating the implications of firm-level responses

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for macroeconomic transitions, a subject that is equally puzzling from the point of view of neoclassical growth theory and the experience of miracle economies[1]. How are macroeconomic variables affected by firm-level adjustments in growth rates and scales of operations in response to forces that promote economic growth? Is the consideration of these margins important for the dynamics of output, productivity, and investment along development paths? The goal of this paper is to address this questions in the context of a dynamic general equilibrium model of entrepreneurship and innovation.

Our baseline economy builds on Lucas (1978), which we extend to incorporate a theory of innovation along the lines of Atkeson and Burstein (2010). There is a large household populated by a continuum of individuals, who are heterogeneous with respect to the ability to operate a firm. Entrepreneurial ability evolves endogenously as a result of entrepreneurs' investments in innovation, and exogenously as a result of productivity shocks. For workers, the arrival of entrepreneurial ideas is completely random. Agents commit to a risk-sharing agreement that insulates them from idiosyncratic fluctuations in income. The head of household, then, makes an occupational choice on behalf of its members, determining whether they work for a wage or operate a business, and makes decisions about aggregate consumption and physical capital accumulation. We construct a low income stationary equilibrium introducing idiosyncratic revenue taxes and subsidies, in the spirit of Hsieh and Klenow (2009) and Restuccia and Rogerson (2008). We trigger development through a reform that liberalizes the economy from all frictions.

To highlight the importance of the endogenous innovation decision and the selection in and out of entrepreneurship, we consider two alternative version of our theory that abstract from one of these key mechanisms.

We first explore the implications of our economy for microeconomic and macroeconomic outcomes in the frictionless and distorted stationary equilibriums. We feed a distribution of distortions that, as in the data, creates a positive correlation between marginal revenue products and physical productivity. We test the implications of the model with respect to its predictions about the average firm size, and the life cycle of firms. We find that the theory can account well for these aspects of the economies of low income countries. We show that innovation is discouraged and entrepreneurship is fostered in presence of distortions, forces that together with resource misallocation contribute to reducing the average firm size. The reduction in innovation incentives leads to a flat growth path over the life cycle of a firm. After 60 years, the average entrant in the distorted economy is a third of the size of its counterpart in the frictionless equilibrium.

At the aggregate level, we find that our calibrated profile of distortions generates a reduction in aggregate productivity (TFP) of 34% and contractions in output of 40%, when combined with the reduction in the capital stock. However, our analysis shows that when compared with the TFP and output losses from an economy with exogenous

[1]See literature review below for an account of existing work documenting the failure of the neoclassical model to account for observed development paths

innovation, the magnification created by the endogenous technology choice is small, amounting to an extra 4% contraction in TFP. We argue this is a small amount because, at the firm-level, the differences between the two economies are orders of magnitude larger. For example, average entrepreneurial ability goes down by 80% with endogenous innovation, and only 20% with exogenous productivity. We show that a countervailing force that mitigates the productivity loss is the response in the number of entrepreneurs, which goes up by a larger amount with endogenous innovation than without.

With respect to development dynamics, on the other hand, the innovation and occupational choice margins play an essential role in reconciling the model with various features of growth accelerations in miracle economies. Upon a reform that eliminates distortions, firms respond along three margins of adjustment: occupational choices, innovation decisions, and factor reallocation. We show that the interaction between innovation, entry, and exit decisions of entrepreneurs, and capital accumulation of the household, yields a sustained and protracted increase in output and TFP, and a hump-shaped path for investment and the interest rate. These patterns are consistent with the type of development dynamics that are observed in the data, and are in opposition to the predictions of the standard neoclassical growth model.

The number of entrepreneurs goes down upon reform, and continues to do so along the transition path to the steady state. The initial decline is explained by the withdrawal of subsidies that artificially protected profits of unproductive entrepreneurs, while the subsequent contraction obeys to the sustained increase in wages, that increases the opportunity cost of entrepreneurship. The increase in the wage is itself the product of the growth in the productivity of succesful incumbents, who innovate more as they now face better prospects for upgrading technologies. The upgrading of technology, however, manifests sluggishly in aggregate productivity, since innovation efforts are risky and the distribution of productivities takes time to reflect the results of firms' innovation efforts. Therefore, the model renders a sustained and protracted increase in TFP.

The decision to increase innovation also has important implications for the dynamics of investment and the rate of return to capital. The latter falls in the early years after the reform, and it overshoots in the intermediate periods before converging to the steady state. Even though distortions were removed, and capital is being reallocated towards highly productive entrepreneurs, there are still very few of those in the economy. Furthermore, the number of agents involved in entrepreneurship has gone down. Thus, capital demand is low. However, as the productivity distribution starts to reflect the increase in the growth rate of firm level productivities, capital demand picks up and so does the rental rate. The dynamics of investment follow that of the interest rate, inhering its hump shape.

Had we abstracted from innovation, entry and exit decisions of entrepreneurs, the behavior our the economy along the development path would have come back to featuring the same counter-factual features of the neoclassical growth model. To show this, we construct a long run equilibrium with and without distortions in which either the evolution of the productivity of entrepreneurs or the occupational choices are exogenous, and explore transition dynamics from the same type of reform as in the benchmark. We find that TFP converges almost immediately in this case, since capital and labor reallocate promptly to their most efficient use. As a result, GDP convergence is significantly faster. In terms of investment and interest rates, the rates of return to capital are the highest upon reform, when the capital stock is low and TFP has jumped up. Investment dynamics, then, take advantage of this shape, spike up upon reform and converge monotonically from above to the new steady state. We conclude from this analysis that, even though the consideration of life-cycle dynamics of firms contributes only mildly to understanding aggregate long run losses from misallocation, it plays an essential role when it comes to understanding development dynamics.

2 Related Literature

Our study provides a unified framework to think about the long run implications of allocative distortions, and to investigate the micro and macro behavior of the economy along development paths. It is therefore related to the large body of studies that has made contributions to each of these areas in isolation.

We find motivation in the burgeoning empirical literature on misallocation and productivity. Several studies have documented the divergence in the allocation of production factors across firms between developed and developed countries. Hsieh and Klenow (2009) and Bartlesman, Haltiwanger, and Scarpetta (2008) are two salient examples. From this literature we adopt the fact that misallocation is a pervasive feature of the economies of low income countries, and require that our theory's initial steady state for the analysis of transitions is consistent with statistical properties of the distribution of marginal revenue products and the firm size distribution in developing economies.

Our work also builds upon a related empirical literature investigating differences in life-cycle dynamics of firms across countries. Hsieh and Klenow (2012) finds that a typical 40 year old manufacturing firm in Mexico is twice as large as it was at entry, while firms display virtually no growth over the life-cycle in India. In the US, on the other hand, a typical entrant multiplies its size by a factor of 8 once it reaches the 40 years mark. Ayyagari et.al. (2013) also find evidence of slower firm growth in a broader sample of developing countries, although these differences are less pronounced than those in Hsieh and Klenow (2012). We use these facts as motivation to bring an endogenous mechanism of firm growth into our model that connects life-cycle dynamics to distortions affecting allocative efficiency. Unlike the cross-sectional implications of the distortions, which will be calibrated to data, we shall explore the life-cycle implications of the model as a test to our theory of innovation.

The quantification of the long run aggregate implications of allocative distortions, and the assessment of the magnification effects created by the endogenous responses in occupational choices and innovation is a subject that is also present in the literature just described. Hsieh and Klenow (2012) finds that accounting for differences in life cycle dynamics of firms across countries can generate productivity loses of up to 25%. Our work relates to this study in two ways. First, it offers an alternative quantification of the steady-state productivity effects of life cycle considerations in presence of distortions, through the lens of a slightly different model. Under our calibration, we find that the endogenous response in innovation efforts by firms magnifies the long run decline in TFP relative to an economy with exogenous productivity growth by a moderate 5%. However, we differentiate from the existing literature in showing that, albeit of lesser importance for long run aggregate outcomes, accounting for endogenous innovation responses by firms plays a critical role for development dynamics.

The second feature of our work connects us with the literature on neoclassical transition dynamics. Christiano (1989) and King and Rebelo (1993) were the first to emphasize the shortcoming of the frictionless neoclassical model when it came to reproducing features of transition dynamics in miracle economies. In the data, transition dynamics of fast growing economies are characterized by sustained growth in income per-capita and total factor productivity, delayed but protracted investment surges, and hump-shaped interest rate dynamics. These features cannot be jointly reproduced by the various extensions to the canonical neoclassical growth model. Our contribution is to propose a model with richer micro-economic underpinnings, disciplined by firm-level data in developed and developing countries, and show that the richer version can successfully reproduce all features of the data.

Two papers in the literature stand out for the proximity with ours. Buera and Shin (2013) develops a theory of transitions featuring heterogeneous entrepreneurs, entry and exit to production, and credit market imperfections. Motivated by the experience of seven Asian economies, the authors show that in presence of financial frictions that delay capital reallocation, transition paths triggered by the removal of idiosyncratic distortions are characterized by investment and interest rate dynamics that are close to the data. The model also yields an endogenous path for TFP, although on this front the model's convergence is faster than in the data. Our relationship to this paper is twofold. On one hand, we take the paper's historical accounts of growth accelerations in fast growing economies as providing empirical support to the idea that reforms that removes allocative distortions occurred at the beginning of these growth accelerations. Secondly, our model provides a complementary mechanism through which macroeconomic dynamics can depart from those of the standard neoclassical model. Rather than emphasizing frictions to factor reallocation, we show that the interaction between the economy's incentives to accumulate tangible capital, through household's investment decisions, and intangible capital, from firms' innovation efforts, can generate transition paths for output, investment, and TFP similar to those in the data in a frictionless setup. Furthermore, because innovation outcomes are risky, the productivity distribution of firms manifests sluggishly the increased innovation efforts by firms, which allows the model to generate sustained and protracted increases in TFP, a weakness of the theories based on barriers to factor reallocation.

The consideration of tangible and intangible forms of capital relates our paper to the work of Atkeson and Kehoe (2005). The authors develop a theory of development in which life-cycle dynamics are driven by age-dependent stochastic accumulation of organizational capital and in which entering firms embody the best available technology. The trigger of development in their model stems from a sudden permanent improvement in the technologies embodied in new plants. Despite the resemblance of our model to theirs, there are several points of departure. First, as in the data, life-cycle dynamics of firms in the frictionless steady state of our model are different from those of the distorted equilibrium. In turn, this differences are generated endogenously, from a theory of innovation that connects firm growth to allocative frictions. Secondly, the predictions about entry along the transition path in our model differ from those in Atkeson and Kehoe (2005). Entry is inefficiently encouraged by subsidies in the pre-reform steady state of our economy, which implies that our development paths are characterized by reductions in entrepreneurship, and increases in the average firm size. Lastly, because of our focus on growth accelerations, we follow a different strategy for parameterizing the pre-reform stationary equilibrium, appealing to firm-level data in low income countries to discipline the choice of distortions that hinder output and productivity.

3 Model

We propose an economy populated by a single household, composed of a continuum of agents. These agents are heterogeneous with respect to their ability to operate a production technology and run a business. The head of household makes an occupational choice on behalf of each agent, choosing either to assign her to entrepreneurship and earn a risky profit, or make her participate in the labor force, in exchange for a fixed wage. Each individual commits to participate in a risk-sharing agreement that insulates individual consumption from fluctuations in idiosyncratic income. In addition to occupational choices, the head of household chooses aggregate consumption and investment in order to maximize lifetime utility.

There are endogenous and exogenous forces for firm dynamics and resource reallocation. The endogenous component stems from entrepreneur's investments in a risky innovation technology that controls the expected evolution of entrepreneurial ability over time. The exogenous element results from idiosyncratic productivity shocks around the expected path. It is the endogenous decision of entrepreneurs to innovate together with the decision to enter and exit entrepreneurship that connects the life cycle and the size distribution of firms with policies and distortions to factor allocation.

3.1 Consumption and Savings Problem

The assumption of perfect sharing of idiosyncratic risk allows us to separate the consumption/investment decision from the choices about occupations.

Taking wages and occupational choices as given, the household chooses consumption and investment in order to solve the following problem:

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to

$$c_t + k_{t+1} = w_t L_t^s + \Pi_t + (1 + r_t) k_t.$$

Aggregate labor supply and aggregate profits, L_t^s and Π_t respectively, are defined as follows:

$$L_t^s = \int (1 - o_t(z)) \, dM_t(z)$$

and

$$\Pi_t = \int o_t(z) \, \pi_t(z) \, dM_t(z)$$

where $o_t(z)$ is the outcome of the occupational choice of a household member with productivity z, being equal to 0 if she is a worker, and 1 is she is an entrepreneur; and $M_t(z)$ denotes the endogenous distribution of agents over productivity levels. All these objects will be characterized in detail below.

3.2 Occupational Choice

We assume that the head of household chooses occupations for its members every period. Furthermore, we assume that movements in and out of entrepreneurship are costless. Therefore, the decision to allocate an individual into working for a wage or becoming an entrepreneur amounts to comparing the values associated with each activity.

When selected into entrepreneurship, agents produce the final good combining their own idiosyncratic productivity, z, together with capital and labor into a Cobb-Douglas production function with decreasing returns to scale[2]:

$$y_t(z) = z^{(1-\alpha-\theta)} k_t(z)^{\alpha} \, l_t(z)^{\theta}$$

We assume that there are perfectly flexible labor and capital rental markets every period, so that both capital and labor can be adjusted freely in response to changes in aggregate or idiosyncratic conditions. It follows that capital and labor choices are determined by the following static maximization problem:

$$\pi_t(z) = \max_{l,k} \left\{ z^{(1-\alpha-\theta)} k^{\alpha} l^{\theta} - w_t l - (r_t + \delta) k \right\}$$

[2]The introduction of the productivity term raised to the $(1 - \alpha - \theta)$ power is a normalization that simplifies the description of the stochastic process for productivity. As we will show below, firms' capital and labor demands become proportional to z when productivity is introduced in this way in the production function. This allows us to map the space of productivity levels z directly into the space of labor and capital demands.

which yields the following expressions for optimal capital and labor demands:

$$l_t(z) = \left(\frac{\alpha}{r_t+\delta}\right)^{\frac{\alpha}{1-\alpha-\theta}} \left(\frac{\theta}{w_t}\right)^{\frac{1-\alpha}{1-\alpha-\theta}} z$$

$$k_t(z) = \left(\frac{\alpha}{r_t+\delta}\right)^{\frac{1-\theta}{1-\alpha-\theta}} \left(\frac{\theta}{w_t}\right)^{\frac{\theta}{1-\alpha-\theta}} z$$

The indirect profit function associated with optimal capital and labor demands is given by:

$$\pi_t(z) = \left(\frac{\alpha}{r_t+\delta}\right)^{\frac{\alpha}{1-\alpha-\theta}} \left(\frac{\theta}{w_t}\right)^{\frac{\theta}{1-\alpha-\theta}} (1-\alpha-\theta)\, z$$

Besides production decisions, entrepreneurs make investments in innovation. We adopt a process of technology upgrading and downgrading similar to that in Atkeson and Burstein (2010). Specifically, we assume that the growth rate of idiosyncratic productivity is normally distributed, with an expected rate of growth that is determined from the firm's investments in innovation, and an exogenous standard deviation.

Let Δ denote the change in the logarithm of productivity that a firm can experience from one period to the other. Entrepreneurs count with a research technology that yields a probability p of a technological upgrade (and probability $1-p$ of a downgrade) in return to investing $\chi(p,z)$ units of labor. We assume a convex function for the cost of innovation of the following form:

$$\chi_t(p,z) = z \times \mu e^{\phi p}$$

Notice that the innovation cost is scaled by the current productivity of the entrepreneur. As we will explain below, this is an important assumption that allows the model to be consistent with innovation patterns of large firms in the U.S, which is our target economy for the calibration. We will also explain the relevance of the scale parameter μ and the elasticity parameter ϕ to replicate of properties of the size distribution and firm life-cycle dynamics in the U.S.[3]

Taking capital and labor demands from the static profit maximization problem, entrepreneurs' innovation decision solves the following optimization problem:

$$\begin{aligned} v_t^E(z) \;=\; max_p \Big\{ & \pi_t(z) - w_t \chi(p,z) \\ & + \left(\frac{1}{1+r_{t+1}}\right) \left[p v_{t+1}\left(ze^{\Delta}\right) + (1-p)\, v_{t+1}\left(ze^{-\Delta}\right)\right] \Big\} \end{aligned}$$

with $v_t^E(z)$ standing for the value of an entrepreneur with productivity z in period t, and $v_t(z)$ denoting the value of an individual in period t with productivity z, facing

[3]We can also describe the process for idiosyncratic productivity as following binomial approximation to a geometric Brownian motion, with an exogenous standard deviation Δ, and endogenous drift $(2p_t(z)-1)\,\Delta$

the decision to become an entrepreneur or working for a wage. We will come back to this value below, once we characterize the value of a worker.

Unlike entrepreneurs, we abstract from modeling workers' efforts and investments in developing entrepreneurial ability. We assume that while working for a wage, agents get a random draw of entrepreneurial ability from a known stationary distribution $F(z)$ that they can exploit the following period if they find profitable to do so. In particular, we assume that an individual in the labor force with current entrepreneurial ability z gets to keep it for the following period with probability ψ, and gets a random draw from the distribution $F(z)$ with probability $(1-\psi)$. The same process governs the evolution of entrepreneurial ability of agents that join the labor force after having exited from operating a business. These agents will keep the accumulated stock of knowledge with probability ψ, and will get random draws with probability $(1-\psi)$.

Our probabilistic representation of the arrival of entrepreneurial ideas among workers allow us to be consistent with two key properties about the behavior of entrants in the data: 1) the rate of establishment entry and exit, and 2) the average size of entrants relative to incumbents. We will see below that consistency with these facts is important for the properties of firm's life-cycle dynamics, and for evaluating the aggregate consequences of allocative distortions.

It follows from the above that the value of a worker is simply defined by the wage rate in the period, plus the discounted expected value of resetting occupations in the following period:

$$v_t^{\omega}(z) = w_t + \left(\frac{1}{1+r_{t+1}}\right)\left[\psi v_{t+1}(z) + (1-\psi)\int v_{t+1}(z')\,dF(z')\right]$$

with the value of an agent before making an occupational choice given by

$$v_t(z) = \max{}_{o_t(z)} o_t(z)\, v_t^E(z) + (1-o_t(z))\, v_t^{\omega}(z)\,.$$

3.2.1 Aggregation and Definition of Equilibrium

At any given point in time, all individuals in the economy will be distributed over the space of entrepreneurial productivities. We denote the fraction of individuals with productivity less than or equal to z with $M_t(z)$. We need to characterize the evolution of such distribution in order to be able to aggregate individual decisions and compute equilibrium prices.

Say we start with a given distribution $M_t(z)$ at the beginning of period t. Entrepreneurs move across productivity levels in accordance to their innovation decisions, while workers do so in response to the stochastic process of productivity. Combining these processes leads to the following law of motion for the distribution of agents across productivity levels:

$$M_{t+1}(z) = M_t(z) + \int_z^{ze^{\Delta}} (1-p_t(x))\, o_t(x)\, dM_t(x) - \int_{ze^{-\Delta}}^{z} p_t(x)\, o_t(x)\, dM_t(x)$$

$$- (1-\psi) \int_0^z (1 - o_t(x))\, dM_t(x)$$

$$+ (1-\psi) F(z) \int_0^\infty (1 - o_t(x))\, dM_t(x) \tag{1}$$

The second two terms refer to the individuals that worked as entrepreneurs in period t and transition to (remain in) the set with productivity in $[0, z]$ after a period. Those with productivity level $x \in \left(z, ze^{\Delta}\right]$ downgrade to $xe^{-\Delta} < z$ with probability $1 - p_t(x)$, and those with productivity level $x \in \left(ze^{-\Delta}, z\right]$ upgrade to $xe^{\Delta} > z$ with probability $p_t(x)$. The last two terms refer to workers. A fraction $1-\psi$ of workers with ability less than z get a new productivity. Among all the workers that get a new productivity, a fraction $(1-\psi)F(z)$ their new draw is less than or equal to z.

A *competitive equilibrium* in this economy is given by sequences of choices by the head of the household $\{c_t, k_{t+1}, o_t(z)\}_{t=0}^{\infty}$; sequences of entrepreneurs' decisions $\{l_t(z), k_t(z), p_t(z)\}$; sequences of interest rate and wage rates $\{r_t, w_t\}$; and a distribution of agents over productivities $\{M_t(z)\}$; such that given an initial capital stock K_0 and a given distribution of talent draws for workers $F(z)$, household's and firm's decision solve their dynamic optimization problems and capital and labor markets clear

$$\int \left[l_t(z) + z^{\frac{1}{1-\alpha-\theta}} h e^{\phi q_t(z)}\right] o_t(z)\, dM_t(z) = \int (1 - o_t(z))\, dM_t(z)$$

and

$$\int k_t(z)\, o_t(z)\, dM_t(z) = K_t,$$

and the distribution of entrepreneurial productivity evolves according to (1).

Similarly, a *long run equilibrium* of this economy is one where individual decisions, aggregate quantitites, and prices are constant, and the distribution of productivities are stationary.

Output and Productivity A well known property of our model with decreasing returns to scale and frictionless factor markets is that the production side of the economy aggregates into the following aggregate production function:

$$Y_t = \left[\int o_t(z)\, z dM_t(z)\right]^{(1-\alpha-\theta)} (K_t^s)^{\alpha} \left(L_{p,t}^s\right)^{\theta}$$

where $L_{p,t}$ stands for aggregate labor demands for the production of the final good only:

$$L_{p,t} = \int l_t(z)\, o_t(z)\, dM_t(z)$$

Measured TFP, in turn, can be computed from the following expression:

$$TFP = \left[\int o_t(z)\, z dM_t(z)\right]^{(1-\alpha-\theta)} \frac{(K_t^s)^{\alpha} \left(L_{p,t}^s\right)^{\theta}}{(L_t)^{1-\alpha}}$$

Notice that we have made an adjustment to our measure of TFP so as to make it comparable with those of income accounting studies. The expression reflects that output is deflated using the entire labor force in the data, regardless of occupations, while in the model only a subset of the agents are involved in the production of goods. The other fraction, entrepreneurs and workers in innovation, make intangible contributions that go unmeasured in GDP.

3.3 Introducing Distortions

One of the goals of this paper is to assess the ability of our theory of innovation and entrepreneurship to generate life-cycle and cross-sectional implications that are consistent with those observed in under-developed countries. To this end, we construct a low income and low TFP stationary equilibrium by means of introducing idiosyncratic taxes and subsidies to the revenues of the firms.

Idiosyncratic distortions create a wedge between the firm's marginal revenue product of labor and capital and factor prices that translate into a reduction in output and TFP. We specify a log-linear relationship between the underlying physical productivity of the firm and the idiosyncratic distortion. We then discipline the slope of this log-linear relationship using information about the correlation between the distribution of marginal revenue products and the the distribution of physical productivities in developing countries [4]. We impose a lump sum transfer to the household to balance the budget of the government.

Formally, let τ_z be the revenue tax or subsidy rate corresponding to a firm with productivity z. Then, distortions and productivity are related in the following fashion

$$[1-\tau(z)] = \left(\frac{z}{z_I}\right)^{-\upsilon(1-\alpha-\theta)}$$

where z_I is the index of the productivity level that we choose to separate firms into those that get a subsidy ($\tau(z) < 0$) and those that are taxed ($\tau(z) > 0$).

Faced with a distortion, the entrepreneur's static profit maximization problem is now given by the following:

$$\pi_t(z) = \max{}_{l,k}\left\{(1-\tau(z))\, z^{(1-\alpha-\theta)} k^{\alpha} l^{\theta} - w_t l - (r_t+\delta)\, k\right\}$$

which yields the following expressions for optimal capital and labor demands:

$$l_t(z) = \left(\frac{\alpha}{r_t+\delta}\right)^{\frac{\alpha}{1-\alpha-\theta}} \left(\frac{\theta}{w_t}\right)^{\frac{1-\alpha}{1-\alpha-\theta}} z\,(1-\tau(z))^{\frac{1}{1-\alpha-\theta}}$$

[4]It is well known from the work of Hsieh and Klenow (2009) and Restuccia and Rogerson (2008), among others, that both the dispersion of marginal revenue products, and its correlation with the distribution of physical productivity, are important statistics for the determination of productivity losses associated with misallocation.

$$\pi_t(z) = \left(\frac{\alpha}{r_t+\delta}\right)^{\frac{\alpha}{1-\alpha-\theta}} \left(\frac{\theta}{w_t}\right)^{\frac{\theta}{1-\alpha-\theta}} (1-\alpha-\theta)\, z\, (1-\tau(z))^{\frac{1}{1-\alpha-\theta}}$$

and the following expression for output and measured aggregate productivity

$$Y_t = \frac{\left[\int z\,(1-\tau(z))^{\frac{\alpha+\theta}{1-\alpha-\theta}}\, o_t(z)\, dM_t(z)\right]}{\left[\int z\,(1-\tau(z))^{\frac{1}{1-\alpha-\theta}}\, o_t(z)\, dM_t(z)\right]^{\alpha+\theta}} (K_t^s)^{\alpha} \left(L_{p,t}^s\right)^{\theta} \tag{2}$$

and

$$TFP = \frac{\left[\int z\,(1-\tau(z))^{\frac{\alpha+\theta}{1-\alpha-\theta}}\, o_t(z)\, dM_t(z)\right]}{\left[\int z\,(1-\tau(z))^{\frac{1}{1-\alpha-\theta}}\, o_t(z)\, dM_t(z)\right]^{\alpha+\theta}} \left(L_{p,t}^s\right)^{\theta} \tag{3}$$

The existence of distortions will also have implications for the entrepreneur's decision to innovate, as reflected by the entrepreneur's value function, which is now given by

$$v_t^E(z) = max_p \left\{ \begin{array}{c} \left(\frac{\alpha}{r_t+\delta}\right)^{\frac{\alpha}{1-\alpha-\theta}} \left(\frac{\theta}{w_t}\right)^{\frac{\theta}{1-\alpha-\theta}} (1-\alpha-\theta)\, z\,(1-\tau(z))^{\frac{1}{1-\alpha-\theta}} - w_t \chi_t(p,z) \\ + \left(\frac{1}{1+r_t}\right) \left[p v_{t+1}\left(ze^{\Delta}\right) + (1-p)\, v_{t+1}\left(ze^{-\Delta}\right)\right] \end{array} \right\}$$

Notice that distortions have no direct effect over the cost function for innovation. Therefore, all the effects of distortions on innovation decisions will operate through their effect over the rate of return to technological upgrade, and general equilibrium effects on labor costs.

4 Quantitative Exploration

In this section we start with the quantitative exploration of our theory. Our first set of results pertain to the long run properties of the model economy. Our goal is to study how agents' occupational choices and innovation decisions are affected by the imposition of allocative distortions, and quantify the aggregate implication of those responses for output and productivity. Even though the distribution of distortions will be parameterized to replicate observed properties of the distribution of marginal revenue products, the model will be tested on its ability to replicate life-cycle dynamics of firms in developing countries, and other statistics of the firm size distribution such as the average firm size. We will argue that the model is able to reproduce the data on these dimensions quite well.

Equipped with a theory that accounts well for firm dynamics in low income countries, we proceed to study the behavior of the economy along transitional dynamics. We trigger development through a reform that eliminates all idiosyncratic distortions. We characterize the dynamics of macroeconomic variables, such us TFP, output, and investment, as well as explore the behavior of the economy in terms of entry and exit rates, innovation, and changes in the firm size distribution over time. We show that our theory accounts well for the slow convergence of output and TFP in the data, as well as the hump-shape dynamics of investments and interest rates. We also show that the innovation, entry and exit margins, which are important to account for both cross-sectional and life-cycle moments, are also essential to generate aggregate dynamics consistent with the data on growth accelerations.

4.1 Calibration

We calibrate parameter values targeting aggregate and firm-level moments of the U.S. economy, and discipline the slope of the log-linear relationship between distortions and productivity using data on the correlation between the marginal revenue products and productivity in developing countries.

For the coefficient of relative risk aversion, we set $\sigma = 1.5$, which is standard in the macroeconomics literature. We set $\beta = 1/(1+0.04)$, to target a 4% yearly interest rate, and set the annual capital depreciation rate at $\delta = 0.06$. In terms of factor shares in the production technologies, given a value of the span of control $1-\alpha-\theta$, we calibrate $\alpha/(\alpha+\theta) = 1/3$, so that 1/3 of the income income going to non-entrepreneurial factors is paid to capital.

The span of control $\alpha + \theta = 0.725$ is calibrated jointly with the parameters of the innovation cost, μ and ϕ, and the innovation step Δ, to match the concentration of income in the top 1% of the population, the tail of the distribution of employment, and the log dispersion of firm growth.

In particular, we assume that entrepreneurial ability shocks are drawn from a discretized Pareto distribution with cumulative distribution function $F(z) = 1 - \left(\frac{z}{z_{min}}\right)^{-\eta}$. We discretize the support over 180 grid points, and we space points so that $log(z_{i+1}) - log(z_i) = \Delta$ for all pair of points in the support. We normalize z_{min} to be equal to 1, and choose the lowest productivity level in the grid to be equal to the $10th$ percentile of the Pareto, and assign a probability mass to each point above the minumum computing $F(z_{i+1}) - F(z_i)$. For a given value of all other parameters, we proceed iteratively over the value of the Pareto parameter η until we match a ratio of average employment of entrants relative to incumbents of 0.2, which is consistent the ratio of the average size of entrants to incumbents in the US manufacturing sector.

For the innovation cost function, we need to specify values for the scale parameter μ and the elasticity parameter ϕ. Both parameters control a single endogenous variable, $p_t(z)$. With a single process for productivity, we therefore can only target one moment in the data. We proceed as follows. For a given value of the elasticity parameter, we calibrate the scale parameter μ to match the slop of the right tail of the size distribution of firms in the US. Luttmer (2010) shows that the tail of the right cumulative distribution function (i.e. one minus the cumulative distribution function) displays a log-linear relationship with respect to employment. The slope of such relationship is 1.06. We adopt this value as the moment to target in the model economy. The parameter ϕ controls the elasticity of innovation effort with respect to changes in innovation incentives. We explore the implications higher and lower innovation elasticities for transitional dynamics when we perform our analysis of reforms. Our benchmark results are for a value of $\phi = 15$, which leads to a calibrated value of $\mu = 0.0000025$.

It remains to be calibrated the size of the productivity jump, Δ. This parameter determines the standard deviation of the growth rate of productivity. Productivity growth rates, in turn, determine employment growth rates in the steady state of a frictionless equilibrium of our model. Therefore, we calibrate $\Delta = 0.25$, which is consistent with the annual standard deviation of employment growth rates for large firms in the U.S.

Lastly, we set the slope of the distortion function to $\upsilon = 0.65$ to generate a TFP drop in the distorted relative to the undistorted stationary equilibrium of 30%, and set z_I so that the aggregate capital to output ratio is unaffected, so as to isolate the effect of idiosyncratic distortions in TFP. The distortions that we impose are also consistent with the relationship between revenue productivity and plant productivity in Mexico and India reported by Hsieh and Klenow (2012).

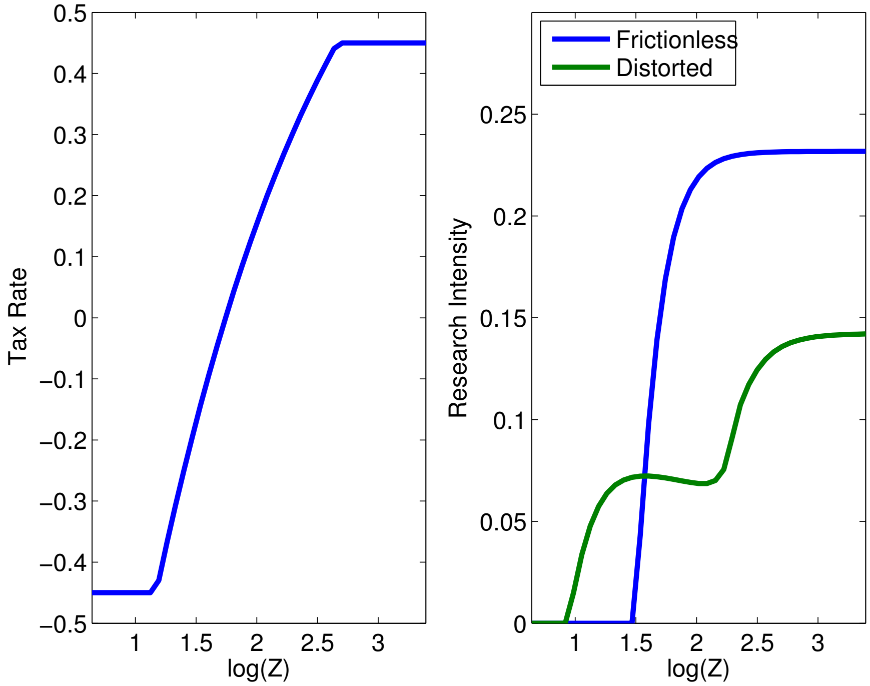
4.2 Long Run Analysis

We start with the analysis of the long run equilibrium of our economy with and without distortions. We first explore the micro-implications of distortions in our theory, as reflected by the distribution of innovation intensities across firms, the distribution of firms across productivities and employment, and the life-cycle dynamics of entrepreneurs. We then compute the macroeconomic effects of these firm-level responses, focusing on the effects on output, TFP, entrepreneurship rates, and average firm sizes.

4.2.1 Micro-Implications

Consider first the effect of idiosyncratic distortions on innovation. The following figure reproduces the schedule of distortions on the left panel, and innovation intensities as a function of productivity in the frictionless and distorted stationary equilibriums on the right one. Specifically, innovation intensity is defined as the ratio of labor demand for innovation relative to labor demand in production.

Figure 1: Distortion Profile and Research Intensities: Frictionless vs Distorted



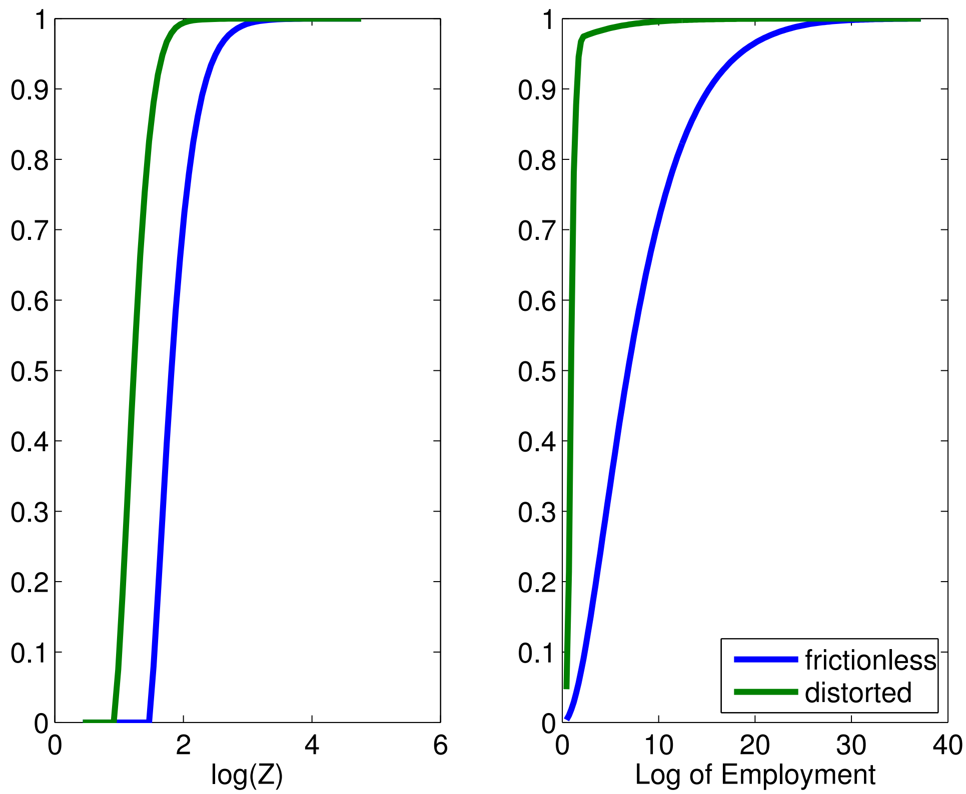
The first salient feature of the graph is that, consistent with Gibralt's law, innovation decisions for sufficiently large firms are independent of productivity, and hence size, in the frictionless economy. This is not surprising, since it is precisely a result of the scaling of the innovation cost function by the firm's productivity. As firms approach the exit threshold, on the other hand, the rate of return to innovation is dampen by the fact that the outside option is independent of the current productivity. That is, close to the exit threshold the downside of not investing in increasing the odds to obtain a succesful innovation are less severe, as unlucky entrepreneurs can exit and work for a wage.

Innovation patterns and occupational choices are significantly different in the distorted economy. First, a larger fraction of agents, with lower entrepreneurial ability, finds it optimal to select into entrepreneurship, taking advantage of the subsidies associated with such activity in the left-end of the productivity distribution. This can be seen by the set of productivities for which innovation intensity was equal to zero in the frictionless economy, and now have a positive value in the distorted one.

Secondly, research intensities in the distorted economy are no longer size independent, not even for large firms. The reason is that the increase in profits allowed by a technological upgrade is down-weighed by a tax that is increasing with productivity. Innovation costs, on the other hand, are not affected by the distortion and scale up with productivity as in the frictionless case. Therefore, rates of return to innovation are undermined relative to its cost. Not only do innovation intensities slope down with productivity for large firms, but the overall level is lower in presence of distortions.

Our theory of innovation also has implications for the productivity and employment distribution across firms. Abstracting from innovation decision can still yield predictions about the employment size distribution purely from reallocation forces. An active innovation channel, however, allows our theory to have predictions about the productivity distribution, besides the properties of the allocation of resources among a given set of technologies. The graph below illustrates the cumulative distribution functions of the number of firms with respect to the log of productivity, and the fraction of employment accounted for by firms with given log-employment levels.

Figure 2: Productivity and Employment Cumulative Distribution Functions



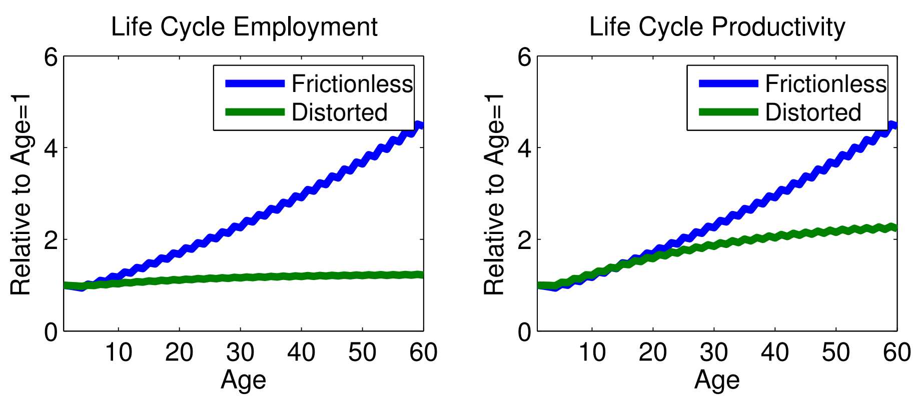
The left panel shows that the distribution of productivities shifts to the left as a result of innovation responses to idiosyncratic distortions. A more remarkable reduction in the mean and standard deviation occurs with respect to the distribution of employment, which compounds the shift in productivities with the effect of the taxes and subsidies. Notice that for our choice of the slope of the distortion profile, almost all of the employment in the distorted economy is accounted for by firms with employment levels less than or equal to the 10-th percentile employment of the frictionless economy. That is, distortions and the consequent response in innovation create a significant concentration of employment in small firms, which is qualitatively consistent with evidence in low income countries.

So far we have shown that our theory is consistent with cross-sectional facts in developing countries once we introduce a particular type of idiosyncratic distortions. However, a distinguishing feature of our economy is that it can be tested along a time-series dimension too, by exploring life cycle dynamics in the frictionless and distorted equilibriums, and comparing them with the data.

Our focus will be on the relative size of establishments after 60 years in one economy against the other, rather than in the relative size of firms with respect to birth in a given environment. As mentioned above, having a single productivity process that is calibrated to match cross-sectional moments of the firms size distribution, we cannot simultaneously replicate the life-cycle patterns of the firms[5]. However, for the purpose of evaluating our theory, we are interested in assessing the extent by which our model of innovation can capture the growth slowdown in productivity growth experienced by firms in developing countries.

The graph below illustrates the expected life-cycle of an entering entrepreneur in a stationary equilibrium with distortions, and in a stationary equilibrium without[6].

Figure 3: Life Cycle Dynamics of Firms



The figure shows a significant slowdown in firm's productivity growth over the life-cycle as a result of allocative distortions. Conditional on survival, a typical 60 year old entrant in the frictionless economy is about three times larger than the typical

[5]This point was initially highlighted by Luttmer (2010)

[6]To generate the average life-cycle of an entrant in the model, we proceeded as follows. Starting off the entrant at the average entrant's productivity, we simulated a large number of trajectories of firm productivity using policy functions for the occupational choice, innovation decision, and productivity shocks. The lines in the graph constitute the average across these simulations.

entrant in the distorted equilibrium. This slowdown is consistent with evidence documented in Hsieh and Klenow (2012), for the cases of India and Mexico, and in Ayyagari, Demiorguc-Kunt, and Maksimovic (2013) for a larger sample of countries.

In summary, our firm-level analysis of the economy in a frictionless and a distorted stationary equilibrium shows that our theory can capture cross-sectional and life-cycle fea‘tures of developing economies where there is evidence of the prevalence of allocative distortions. Our goal now is to study the aggregate implications of these firm-level effects

4.2.2 Aggregate Implications

Our goal now is to understand the aggregate effects of firm-level responses in the model. In order to isolate the contribution of the endogenous innovation channel for long run macroeconomic variables, we compare results against those arising from a version of our model with exogenous expected growth rates. In particular, this version of the model consists of endowing the firms with the same profile of productivity growth probabilities of the endogenous model, without any cost, and keeping the profile unchanged when feeding the same schedule of distortions into the model. Results are displayed in table 4.2.2.

	Endogenous Innovation	Exogenous z (a)	Exogenous z (b)	Exogenous Occupation
GDP	0.62	0.80	0.90	0.70
TFP	0.70	0.85	0.92	0.77
Av. Productivity	0.04	0.50	0.63	0.46
Capital Stock	0.89	0.94	0.97	0.92
# of Entrepr. (diff.)	0.23	0.03	0.14	–
Innovation Labor (diff.)	-0.15	–	–	-0.19

Table 1: Aggregate Variables, Stationary Equilibrium: Endogenous vs Exogenous Productivity/Occupation. For the case with exogenous productivity we present results for two alternative calibrations: (a) Entrepreneurial productivity follows the process in the frictionless case; (b) Productivity follows the process in the distorted case. The values are relative to the frictionless steady state, except for the number of entrepreneurs and the labor demand, which corresponds to the differences between the distorted and frictionless allocations.

	Endogenous Innovation	Exogenous z (a)	(b)	Exogenous Occupation
GDP	0.62	0.80	0.90	0.84
TFP	0.70	0.85	0.92	0.88
Av. Productivity	0.04	0.50	0.63	0.77
Capital Stock	0.89	0.94	0.97	0.99
# of Entrepr. (diff.)	0.23	0.03	0.14	–
Innovation Labor (diff.)	-0.15	–	–	-0.05

Table 2: Aggregate Variables, Stationary Equilibrium: Endogenous vs Exogenous Productivity/Occupation. For the case with exogenous productivity we present results for two alternative calibrations: (a) Entrepreneurial productivity follows the process in the frictionless case; (b) Productivity follows the process in the distorted case. The values are relative to the frictionless steady state, except for the number of entrepreneurs and the labor demand, which corresponds to the differences between the distorted and frictionless allocations.

The results in the table validate some of the findings in the existing literature on misallocation and development. Allocative distortions can cause substantial loses in aggregate productivity, of around 30 to 35 %, and output loses that amount to up to 40% taking into account the effect of distortions on the economy's capital stock.

The distorted allocation is also characterized by a significant increase in the number of entrepreneurs, which helps to mitigate the aggregate loss in productivity and output. The increase in entrepreneurship is partly explained by the subsidization of establishments in the lower end of the productivity distribution, and partly by a decrease in wages, which reduces the opportunity cost of entrepreneurship. Notice that entrepreneurship increases proportionally more in the endogenous innovation economy than in the exogenous innovation one. This is because with endogenous innovation, there is a further reduction in labor demand, and hence wages, due to the contraction in innovation intensities. By reducing the opportunity cost of entrepreneurship, it increases the number of agents that engage in business operations. The stronger increase in the number of entrepreneurs explains why, in spite of a significant reduction in the average talent of entrepreneurs, aggregate TFP loses are only 4% larger with endogenous than with exogenous innovation.

A salient feature of our results, which is a prominent feature of the data, is the reduction in the average size of a typical entrepreneur in the distorted allocation. This occurs both in the exogenous innovation case, as a consequence of the increase in the number of entrepreneurs. This is reinforced in the endogenous innovation model, as a result of the contraction in the average entrepreneurial ability.

In summary, our results suggest that even though accounting for firms' responses in innovation efforts lead to significant changes in firms' life cycle dynamics, and the productivity and size distribution of firms, these changes boil down to a mild extra 5% productivity loss due to allocative distortions in the aggregate. However, in the next section we will show that consideration of the innovation channel plays an essential role for understanding macroeconomic transitions.

	Endogenous Innovation	Exogenous z (a)	(b)	Exogenous Occupation
Av. Productivity	0.41	0.82	0.88	0.80
# of Entrepr.	1.87	1.22	1.20	–
Misallocation	0.95	0.86	0.95	0.85
Production Labor	0.95	0.98	0.92	1.12

Table 3: Decomposition of the Relative Productivity Differences: Endogenous vs Exogenous Productivity/Occupation. For the case with exogenous productivity we present results for two alternative calibrations: (a) Entrepreneurial productivity follows the process in the frictionless case; (b) Productivity follows the process in the distorted case. The values are relative to the frictionless steady state. The product of the terms in each column give the value for TFP in Table 4.2.2. See the Appendix for a discussion of the different terms in this decomposition.

	Endogenous Innovation	Exogenous z (a)	(b)	Exogenous Occupation
Av. Productivity	0.41	0.82	0.88	0.93
# of Entrepr.	1.87	1.22	1.20	–
Misallocation	0.95	0.86	0.95	0.91
Production Labor	0.95	0.98	0.92	1.03

Table 4: Decomposition of the Relative Productivity Differences: Endogenous vs Exogenous Productivity/Occupation. For the case with exogenous productivity we present results for two alternative calibrations: (a) Entrepreneurial productivity follows the process in the frictionless case; (b) Productivity follows the process in the distorted case. The values are relative to the frictionless steady state. The product of the terms in each column give the value for TFP in Table 4.2.2. See the Appendix for a discussion of the different terms in this decomposition.

4.3 Development Dynamics

Equipped with a theory that can account for salient features of firm dynamics in low income countries, we proceed to evaluate the implications of the theory for development dynamics. It is a well known weakness of the standard neoclassical growth model that it cannot simultaneously account for all of the salient features of development dynamics in the data: sustained increases in TFP, hump-shaped dynamics for investment and interest rates. Our goal is to provide a frictionless explanation of this macroeconomic behavior studying the interaction between occupational choices, innovation, reallocation and physical capital reallocation. To accomplish this goal, we follow Buera and Shin (2013), in thinking of development as triggered by a reform that permanently and credibly eliminates all taxes and subsidies in the economy.

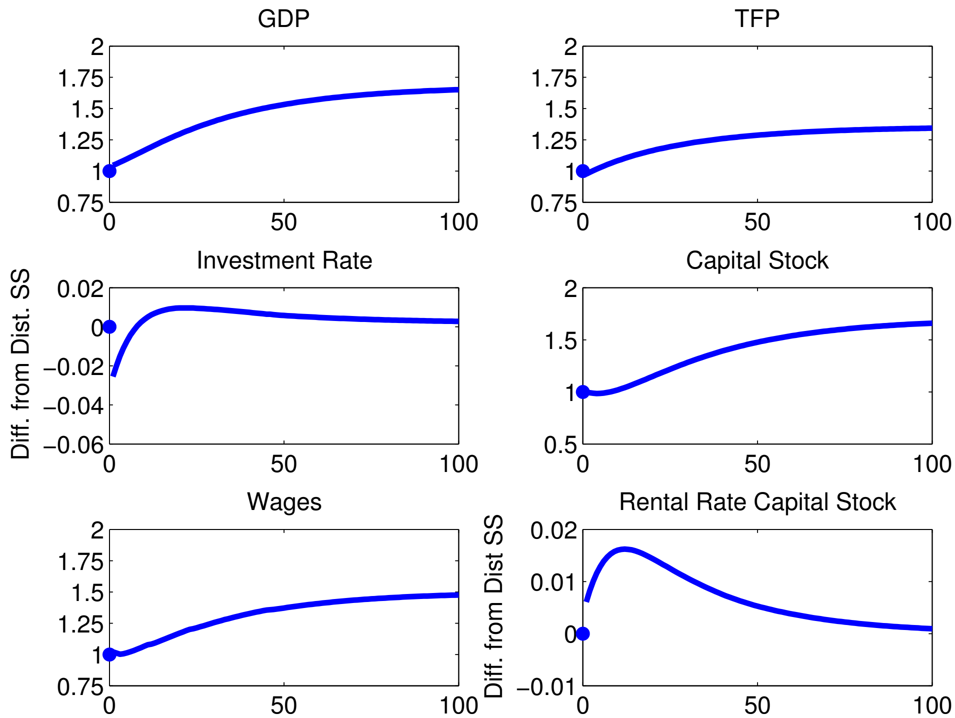


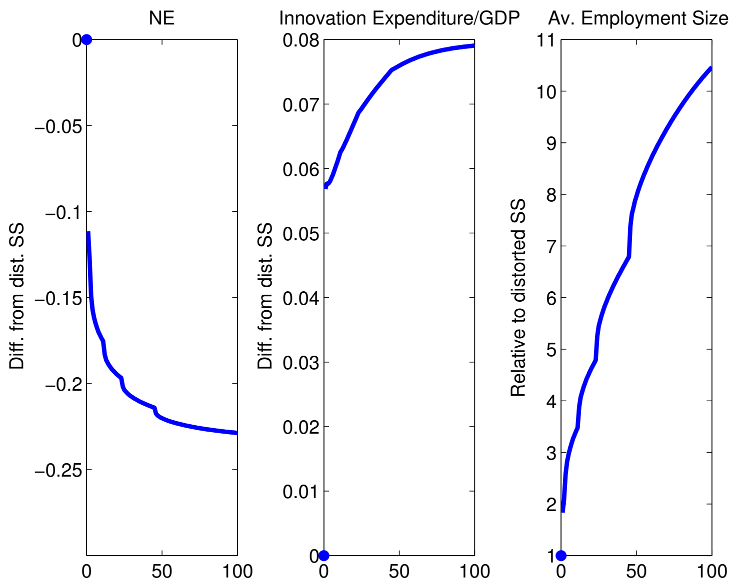
Figure 4: Development Dynamics, Macroeconomic Variables. All variables expressed relative to value in the distorted stationary equilibrium, except for the rental rate of capital, which is plotted as differences.

Figure 4.3 reproduces macroeconomic variables along transition dynamics in our benchmark economy. The figure shows the ability of the model to generate a sustained and protracted increase in output, explained in part by a sustained increase in TFP. Since the economy is free of restrictions to factor reallocation, the delayed response in productivity is entirely driven by the slow response in the distribution of firms over productivities, which takes time to reflect the change in firms' incentives to innovate.

The investment rate is hump-shaped, and leads to a non-monotone path for the economy's capital stock. The capital stock declines in the initial years after the reform, before it recovers and converges to its frictionless steady state level. This is due to the economy's desire to increase the share of resources allocated to innovation activities. Recall that innovation is a labor intensive activity, and that labor is in excess supply after the shock, due to a reduction in the number of entrepreneurs. The reduction in the wage together with an increase in the rate of return to innovation, specially by large firms which were previously affected by a tax, justify that the economy devotes more resources to this purpose.

The incentive to postpone capital investment is also reflected in the dynamics of the interest rate. The rate of return to capital falls in the early years after the reform, and it overshoots in the intermediate periods before converging to the steady state. Even though distortions were removed, and capital is being reallocated towards highly productive entrepreneurs, there are still very few of those in the productivity distribution. Furthermore, the number of agents involved in entrepreneurship has gone down. Thus, capital demand is low. However, as the productivity distribution starts to reflect the increase growth rate in firm level productivities, capital demand picks up and so does its rental rate.

Figure 5: Development Dynamics. Microeconomic Variables



The next figure summarizes the micro-level adjustment of the economy to the reform. It depicts the dynamics of the number of entrepreneurs, the aggregate share of innovation expenditures relative to GDP, and the average employment size in the economy. The number of entrepreneurs declines upon the removal of the distortions, and continues to do so over time. The initial contraction takes place because of the elimination of subsidies that were artificially increasing low productivity firms' profits. This occurs despite the initial decline in wages. Over time, as high productivity entrepreneurs push up the wage rate, entrepreneurship becomes less attractive to a larger fraction of agents, further reducing the number of entrepreneurs along the convergence path.

The middle panel refers to the dynamics of the aggregate share of innovation expenditures relative to GDP in the economy. These are reminiscent of intangible investments the economy makes to improve the growth rate of aggregate productivity. Unlike investments in physical capital, the rate of return to which is typically the highest the lower the physical capital stock, investments in innovation increase monotonically over time. The source of this differential behavior is that innovation intensities are the highest for high productivity firms. Thus, as the participation of high-productivity firms in the economy's productivity distribution increases, it also does the aggregate expenditure devote to innovation activities.

The panel to the right illustrate the dynamics of the economy's average firm size, measured by the size of its productive labor force. All the firm level adjustments taking place in the economy contribute to the increase in the size of a typical production unit. First, labor and capital are being reallocated towards more productive entrepreneurs, which operate larger plants. Secondly, low productivity entrepreneurs are exiting production and becoming workers, which concentrates production in high-ability entrepreneurs even further. Lastly, firms are devoting more resources to upgrading technology levels.

In short, transitional dynamics in our model economy share features of growth accelerations in the data along many dimensions. There is a sustained and protracted increase in TFP, investment and interest rates are hump-shaped, and average firm size increases smoothly over time. These predictions are starkly different from those of the neoclassical growth model, where investment and interest rates spike up upon reform and decrease monotonically over time, and there are no endogenous forces driving productivity and average firm size along the transition path. The differentiation of our model from the standard neoclassical growth theory stems purely from the interaction between innovation incentives post-reform, changes in occupational choices, and their interactions with physical capital accumulation, without any contribution from capital adjustment costs, or frictions to the reallocation of factors across firms. As we shall show below, the endogeneity of innovation decisions is the essential ingredient for the differentiation of our model from the standard neoclassical growth one. As soon as the innovation channel is shut down, transitional dynamics go back to displaying the same shapes as the theory with a representative firm.

4.3.1 Endogenous vs Exogenous Innovation, Entry and Exit

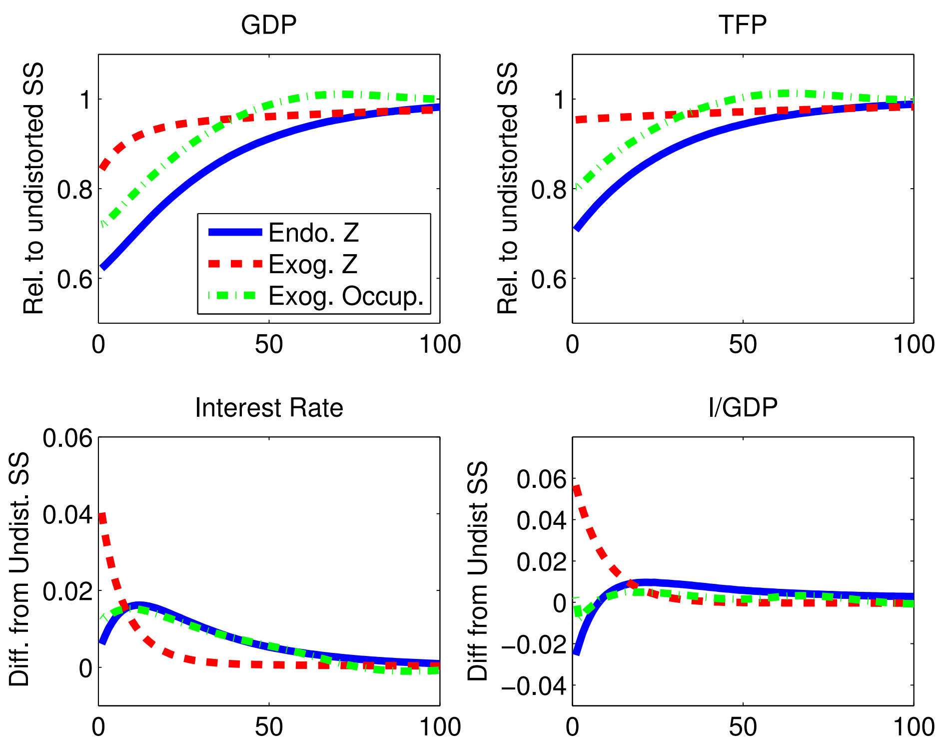
Here we construct two alternative economies that differs from the one we have been considering so far only in that either innovation is exogenous or entry and exit of entrepreneurs are exogenous. As we did when discussion the results for the stationary equilibrium, we use these model variantes to highlight the role of the endogenous innovation, entry and exit margins.

In the first model variant, the probabilities of technological upgrades and downgrades, $p_t(z)$, are set exogenously to the firm. We feed firms with the same innovation profile as the frictionless stationary equilibrium of the endogenous innovation economy. We assume that entrepreneurs that become active only when distortions are in place, adopt the same probability as the least productive entrepreneur of the frictionless equilibrium. In terms of calibrating distortions, we do so in a way that makes the level of income in the distorted stationary equilibrium with exogenous innovation be the same as in the endogenous benchmark. This ensures that the start and end points are the same between the two economies. Innovation probabilities remain unchanged between the frictionless and distorted equilibrium.

In the second model variant we consider the same model of innovation as in the benchmark economy, but assume that the selection in and out of entrepreneurship is exogenous. To obtain an stationary distribution, we assume that 10% of entrepreneurs die each period and are replace by new entrepreneurs. The (constant) fraction of entrepreneurs is set to be equal to that in the frictionless stationary equilibrium of the benchmark economy. All other parameters are as in the benchmark model.

Macroeconomic dynamics for the two alternative model, together with those of the benchmark economy, are illustrated in figure6[7]

Figure 6: Development Dynamics: Endogenous vs Exogenous Innovation/Occupation



The figure makes evident the claim that abstracting for life-cycle considerations brings back all the features of the standard neoclassical growth model to our model of heterogeneous firms and entrepreneurship. TFP converges almost immediately, since all factors reallocate frictionlessly to their most efficient use. As a result, GDP convergence is significantly faster. In terms of investment and interest rates, the rates of return to capital are the highest upon reform, when the capital stock is low and TFP has jumped up. Investment dynamics, then, take advantage of this shape and follows a similar path as the interest rate.

[7]Notice that the current calibration does not quite achieve the same income level in the steady states, which is why variables do not converge yet to the exact same value.

We conclude from the analysis of this section that, even though the consideration of life-cycle dynamics of firms contributed only mildly to understanding aggregate long run losses from misallocation, it plays an essential role when it comes to understanding development dynamics.

5 Conclusion

The standard neoclassical growth model cannot account for the observed features of development dynamics in fast growing economies. In this paper we proposed an extension to the standard framework with richer microeconomic underpinnings disciplined by data on firm behavior in developed and developing countries in order to explore the contributions of these micro-adjustments to overcoming the difficulties of the canonical model.

Motivated by evidence of resource misallocation in low income economies, we introduced idiosyncratic distortions into our model in order to construct a low income equilibrium with cross-sectional implications for the firm size distribution and the distribution of marginal revenue products that are consistent with the data. We then tested our model to show that it delivered factual implications for the life-cycle dynamics of firms, and occupational choices of households.

Equipped with a theory of underdevelopment consistent with micro-data, we then explored the behavior of the economy along a transition growth path triggered by a reform that removed all allocative distortions. We found that once the responses in firms' decision to innovate, the household's occupational choices , and physical capital accumulation decisions are accounted for, the theory can deliver macroeconomic dynamics consistent with the data. We uncovered that the essential ingredient for this improvement lied in the interaction between the incentives to accumulate physical capital and firms' desire to invest in technological improvements. Had the latter feature not been taken into account, transitional dynamics would have reverted to the usual neoclassical patterns, even when preserving the occupational choice margin and the heterogeneity in production.

As argued above, the literature has explore other mechanisms to better understand the forces that drive macroeconomic variables along development paths. The next step in the research agenda is to incorporate those mechanisms, based on barriers to resource reallocation, with the forces highlighted in this paper, in the context of unified framework.

A Decomposition of Aggregate Productivity

In this appendix we discuss the decomposition of the source of TFP differences across stationary equilibria presented in Table 4.2.2. Rearrenging equation (3), we can write

$$\begin{aligned} TFP_t &= \left[\int zo_t(z) \frac{dM_t(z)}{\int o_t(z)\, dM_t(z)}\right]^{1-\alpha-\theta} \\ & \left[\int o_t(z)\, dM_t(z)\right]^{1-\alpha-\theta} \\ & \frac{\frac{\left[\int z(1-\tau(z))^{\frac{\alpha+\theta}{1-\alpha-\theta}} o_t(z) \frac{dM_t(z)}{\int o_t(z) dM_t(z)}\right]}{\left[\int z(1-\tau(z))^{\frac{1}{1-\alpha-\theta}} o_t(z) \frac{dM_t(z)}{\int o_t(z) dM_t(z)}\right]^{\alpha+\theta}}}{\left[\int zo_t(z) \frac{dM_t(z)}{\int o_t(z) dM_t(z)}\right]^{1-\alpha-\theta}} \\ & L_{pt}^{\theta}. \end{aligned}$$

The first line gives the contribution of the average productivity of active entrepreneurs. When entrepreneurs face idiosyncratic distortions which subsidize less productive entrepreneurs, and tax productive ones, this term decreases for two reasons: 1) marginal, less productive individuals become entrepreneurs; 2) active entrepreneurs invest less in innovation, and therefore, the distribution of productivity of active entrepreneurs worsens. The second line gives contribution of the total number of active entrepreneurs. To the extent that there are diminishing return to the variable factors, increasing the number of entrepreneurs while holding fixed their average productivity results in a higher aggregate TFP. The third term gives the (negative) contribution of the misallocation of capital and labor across entrepreneurs. In an undistorted economy this terms equals one. Finally, the last term is the contribution of the labor use in the production. This terms would increase if either the number of entrepreneurs increases, or the labor used to produce innovations increases.

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