Entrepreneurial Risk-Taking, Young Firm Dynamics, and Aggregate Implications*

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Abstract

Despite the importance of high-growth young firms for economic growth, determinants of their growth and survival dynamics are not well understood. In this study, I develop a dynamic occupational choice model that identifies a key predictor of the early growth trajectory of young firms: the outside options of the business founders. I show that entrepreneurs with higher outside options as paid workers tend to take larger business risks, and thus exhibit a more up-or-out type of firm dynamics. I find empirical support for the model's predictions using a large founder-firm matched data set built from administrative databases of the U.S. Census Bureau. I find that controlling for past business performance, young firms operated by entrepreneurs with higher outside options exhibit (i) higher firm exit rates, (ii) more growth dispersion, and (iii) faster growth conditioning on survival. With the calibrated model, I find that deterioration in the outside options of entrepreneurs can have a sizable negative impact on aggregate output and productivity via lower risk-taking by young firms and slower growth in their life cycle. These findings indicate that the expected post-failure outcomes of entrepreneurs are an important factor that governs young firm growth as well as aggregate output and productivity.

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1 Introduction

A long-standing literature in economics, dating back to at least Schumpeter (1942), show that business startups and entrepreneurs play a critical role in innovation, job creation, and productivity growth.[1] Yet, recent studies caution that there is a large heterogeneity in growth dynamics amongst young firms (see, e.g., Schoar, 2010; Hurst and Pugsley, 2011; Decker, Haltiwanger, Jarmin, and Miranda, 2014; Guzman and Stern, 2016). In fact, typical startup firms either exit or exhibit little or no growth, and a small fraction that grow rapidly—so-called high-growth young firms—account for the vast majority of the aggregate contribution of young firms (Decker, Haltiwanger, Jarmin, and Miranda, 2014). However, relatively little is known regarding the economic factors that drive the large differences in the growth dynamics of young firms, and more importantly, the mechanisms through which high-growth young firms are created.

In this paper, I propose a key predictor of the early growth trajectories of young firms: the outside options of the business founders. I argue that startup entrepreneurs with higher levels of outside options, which I define by the level of labor income they expect to earn in the event of business failure, are more likely to take larger business risks and thus exhibit a more up-or-out type of firm dynamics. This is because the option to cease business operations and switch to paid employment serves as insurance against business failure, and better insurance enables individuals to take larger risks. Therefore, entrepreneurs with better outside options are more likely to create high-growth young firms at the cost of a higher failure risk, and those with weaker outside options are more likely to create businesses that stay small.[2]

[1]For models of entrepreneurship and innovation, see, among others, Aghion and Howitt (1992), King and Levine (1993), Acemoglu, Akcigit, Bloom, and Kerr (2013), and Acemoglu and Cao (2015). Haltiwanger, Jarmin, and Miranda (2013) and Decker, Haltiwanger, Jarmin, and Miranda (2014) provide empirical evidence on the importance of young firms in aggregate job creation. For evidence on productivity growth, see Haltiwanger, Jarmin, Kulick, and Miranda (2016) and Alon, Berger, Dent, and Pugsley (2017).

[2]I confine the scope of this study to firms that hire at least one employee and exclude nonemployer self-employment activities. Given that the purpose of this study is to examine diverse firm outcomes including employment growth, I consider this as a fair restriction. However, nonemployer businesses are massive in number and deserve investigation as well. For a recent study on the growth outcomes of nonemployer firms, see Fairlie and Miranda (2017).

To formalize this argument, I construct a dynamic occupational choice model in which individuals can choose between paid employment and entrepreneurship. I build on earlier work by Vereshchagina and Hopenhayn (2009) and model risk-taking by entrepreneurs as the choice of dispersion in the innovation to their business productivity. I refer to this choice as *risky experimentation*. Success in experimentation delivers an increase in business productivity, which translates into growth in firm profits and size. Failure in experimentation results in persistent damage to business productivity, which leads to contraction or even to the exit of the firm. Vereshchagina and Hopenhayn (2009) show that the option to return to paid employment creates a convexity around the exit margin of the objective functions of the entrepreneurs, which endogenously generates risk-taking incentives. I extend and modify their framework by introducing persistence in the firm productivity process and heterogeneity in labor earnings to generate implications on the relationship between the entrepreneurs' outside options and their post-entry firm dynamics.

I begin the analysis by presenting a stylized two-period version of the model to illustrate the mechanism in its simplest form and to derive analytical solutions that can be mapped into empirically testable predictions. The simple model predicts that firms operated by entrepreneurs with better outside options should exhibit (i) higher exit rates, (ii) more growth dispersion, and (iii) faster growth conditioning on survival compared to firms operated by entrepreneurs with weaker outside options, holding lagged firm productivity constant. I show that it is important to control for lagged productivity to uncover the predicted patterns in the data; the unconditional correlations between outside options and firm exit is ambiguous, given the likely positive correlation between outside options and initial business productivity. The model also implies that when an entrepreneur has strong nonpecuniary incentives for being an entrepreneur (e.g., being one's own boss, having a flexibile work schedule), the impact of his outside option on the three predicted outcomes stated above will be mitigated. This is because if all else is equal, he will be more averse to losing the nonpecuniary benefits of staying in entrepreneurship, and therefore will take fewer risks. This result is consistent with evidence documented by Hurst and Pugsley (2011) that startup business owners who report strong nonpecuniary motives also tend to report a lack of willingness to take risks to grow their firms.

I provide direct empirical evidence for the model's predictions using a panel data of 1.7 million startup firms. To test the model's predictions, one needs a data set that provides information on business founders as well as longitudinal records of their firms, including firm productivity. I construct such unique data set by combining individual- and firm-level administrative databases of the U.S. Census Bureau. It not only contains the demographics and work histories (e.g., earnings, workplaces) of the business founders, but also tracks annually each firm from its first year of operation until exit (if it occurs). Because the outside option is not directly observed in the data, I use the business founders' annual labor earnings prior to business entry as a proxy variable for their outside options. This approach is based on empirical evidence that labor earnings prior to business entry is a strong positive predictor of labor earnings post-business exit, especially for short spells of entrepreneurship.[3] I measure firm productivity by revenue per worker, which is a frequently used measure in the firm dynamics literature.

A major concern of this empirical test is that the outside options of entrepreneurs are likely to be positively correlated with unobserved abilities, such as managerial capabilities, which independently have a positive impact on firm growth and survival outcomes. I find that higher outside options predict higher firm exit rates, once I control for lagged firm productivity and size. A large component of unobserved ability should be captured by lagged firm productivity and size, and if any effect is left over, it should create a bias toward finding a negative relationship between outside options and firm exit. In addition, it is unclear outside the proposed model mechanism why better unobserved abilities should lead to a larger growth dispersion.

[3]For example, see Williams (2000) and Bruce and Schuetze (2004). I also report a strong positive correlation between prior and post entrepreneurship earnings for the entrepreneurs that exit. The literature also finds that the effect of past entrepreneurship experience on wages is generally smaller than the effect of experience in paid employment. This feature is reflected in the quantitative model in section 4.

Micro-level theoretical and empirical analyses indicate that the outside options of business founders are important determinants of young firm growth and survival dynamics. Yet, the question remains whether the proposed mechanism have quantitatively meaningful implications for macro-level outcomes such as aggregate output and productivity. To address this question, I embed the stylized model into a heterogeneous agent general equilibrium model and calibrate it to the U.S. economy. I find that a decrease in outside options for startup entrepreneurs can have a sizable impact on aggregate productivity and output. In a counterfactual where the option to return to paid employment is completely removed, aggregate output falls by 8.9%, and aggregate output per worker falls by 4.4%. I find that this result is mainly driven by a reduction in risk-taking by young firms, which results in slower productivity growth along their life cycles. Therefore, outside options are important factors that affect not only young firm growth and survival, but also aggregate output and productivity.

This paper contributes to the entrepreneurship literature that attempts to better understand the gap between a broad population of entrepreneurs with low business growth prospects (e.g., Hamilton, 2000; Hurst and Pugsley, 2011) and a small number of transformative entrepreneurs with strong capabilities and ambition for rapid growth (e.g., Guzman and Stern, 2016; Haltiwanger, Jarmin, Kulick, and Miranda, 2016). In the developing economy context, Schoar (2010) argues that policy interventions which lack a clear understanding of the difference between those two types of entrepreneurs may result in unintended adverse consequences. While the existing studies tend to adopt such dichotomous view on the types of entrepreneurship, I contribute to this literature by identifying outside options as a relatively continuously distributed source of heterogeneity among entrepreneurs.

This paper also contributes to an emerging literature in firm dynamics and macroeconomics that focuses on the determinants of firm entry and growth along their life cycle. Recent empirical studies found that while young firms make substantial contribution to aggregate job creation and productivity growth, the U.S. economy has been experiencing a secular decline in firm entry rates.[4] In addition, recent studies found that there is a tight linkage between life-cycle dynamics of plants and firms and aggregate productivity (e.g., see Hsieh and Klenow, 2014; Akcigit, Alp, and Peters, 2016). These findings triggered interest among macroeconomists in the life-cycle aspects of firm growth, particularly those of young firms.[5] I contribute to this literature by showing that deterioration in the outside options of entrepreneurs result in a decline in firm entry rates and slower life-cycle growth of young firms.

In addition, this paper contributes to the literature that investigates the experimental aspect of entrepreneurship (e.g., Kerr, Nanda, and Rhodes-Kropf, 2014). This literature emphasizes that entrepreneurship should be viewed as an experiment that can be reversed, and that post-failure options should be taken into consideration in analyzing entrepreneurship decisions. Work by Polkovnichenko (2003) and Vereshchagina and Hopenhayn (2009), and more recently by Manso (2016) and Dillon and Stanton (2017), confirms this idea, and demonstrates that the option to return to paid employment can largely explain why some people enter entrepreneurship despite the low risk premium of entrepreneurs relative to wage earners observed in the cross-sectional earnings distribution (Hamilton, 2000). In a similar vein, empirical studies find that providing insurance against failure from entrepreneurship such as job-protected leave (Gottlieb, Townsend, and Xu, 2016), unemployment insurance (Hombert, Schoar, Sraer, and Thesmar, 2017), and cash transfers (Bianchi and Bobba, 2012) spurs entry to entrepreneurship. I contribute to this literature by providing new empirical evidence such that when lagged firm performance is controlled, higher outside options are associated with higher exit rates and a larger growth dispersion. This result is consistent with the experimentation view of entrepreneurship.

Lastly, this paper is closely tied to existing models of firm dynamics with endogenous innovation choices that involve potential risks (e.g., see, among others, Atkeson and Burstein,

[4] For example, see Haltiwanger, Jarmin, and Miranda (2013), Decker, Haltiwanger, Jarmin, and Miranda (2014), Decker, Haltiwanger, Jarmin, and Miranda (2016) and Alon, Berger, Dent, and Pugsley (2017)

[5] For recent examples, see Pugsley and Sahin (2015), Arkolakis, Papageorgiou, and Timoshenko (2017), Sedlácek and Sterk (2017), and Moreira (2017).

2010, Gabler and Poschke, 2013, Caggese, 2016, and Buera and Fattal-Jaef, 2016). I contribute to this literature by showing that modeling heterogeneity in the post-exit value of firms is important in capturing the dynamics of firms near the entry and exit margins. Typical existing models assume that firms face homogeneous post-failure outcomes by specifying the value of exit as a constant, which is typically set at zero, and focus on other frictions or distortions that affect firms' innovation decisions. Some of the existing models, such as that of Caggese (2016), recognize that the existence of an exit option generates extra risk-taking incentive for firms, but rarely go further to specify the source of the exit option. I show that modeling the impact of outside options on firm dynamics is important and that using prior earnings can be one way to discipline the distribution of post-exit values. Capturing the dynamics of firms near the entry and exit margins is important, as these firms include startups and young firms, which play an important role in aggregate growth.

The paper is organized as follows. Section 2 develops a simple two-period single-agent model that illustrates the risk-taking mechanism. Section 3 describes the empirical investigation of the simple model predictions. Section 4 extends the simple model to a quantitative heterogeneous agent general equilibrium model, and Section 5 describes the model calibration and counterfactual exercises, and Section 6 concludes.

2 A Simple Model of Business Risk Taking

In this section, I present a simple two-period single-agent model of business risk-taking. This model formalizes the mechanism of the hypothesis in its simplest form. It generates a set of predictions on the relationship between outside options and firm growth and survival, which are then empirically tested in Section 3. It also serves as a key building block of the quantitative general equilibrium model presented in Section 4.

There are two periods, denoted as $t = 1, 2$. Consider an entrepreneur in $t = 1$ who is endowed with a business idea z_1 and labor efficiency h.[6] For simplicity, it is assumed that

[6] In the simple model, I abstract from entrepreneurship entry decisions and focus on post-entry dynamics.

the agent has log utility and all income is comsumed without saving in each period; these assumptions are relaxed later in the quantitative model. In the first period, he hires effective units of labor n_1 and pays wn_1 to workers, where w is the wage per effective unit of labor. He produces output via production function $y_1 = z_1^{1-\alpha} n_1^{\alpha}$. His next-period business idea z_2 evolves according to a binomial risky innovation process

$$z_2 = \begin{cases} z_1 e^{\Delta} & \text{with probability} \quad e^{-\gamma\Delta} \\ z_1 e^{-\Delta} & \text{with probability} \quad 1 - e^{-\gamma\Delta} \end{cases}$$

where $\Delta \geq 0$ is a choice variable and $\gamma > 0$ is a parameter. Binomial innovation process has been used in existing models of firm dynamics (see, e.g, Atkeson and Burstein, 2010; Buera and Fattal-Jaef, 2016; Caggese, 2016). A key assumption introduced in this model is that Δ can be controlled by entrepreneurs, while Δ has been treated as a fixed parameter in previous models.[7] This assumption enables the model to predict that some types of firms exhibit larger growth dispersion or higher exit rates than others. Hereafter, I refer to choosing a positive Δ as conducting a *risky experimentation*. The success probability $e^{-\gamma\Delta}$ is assumed to be decreasing in Δ, indicating that riskier experiments are more challenging to implement successfully.

This risky experimentation specification can be thought of as representing several real-world business risk-taking choices made in the post-entry phase. For instance, firms can try to adjust their target customer base: A firm that initially targeted a niche market may try to expand to a broader customer base, in which case it could lose its existing customers in the case of failure. Other examples of risk-taking choices include adding or removing

I discuss how outside options affect individuals' entry decisions later in section 5.3 using the quantitative model.

[7] Atkeson and Burstein (2010) and Buera and Fattal-Jaef (2016) assume that firms can increase the success probability subject to an increasing cost function while Δ is a fixed parameter. In Caggese (2016), both success probability and Δ are treated as parameters in which Δ is specified as risky innovation to fixed cost of operation. Gabler and Poschke (2013) study a firm dynamics model with innovation dispersion choice. An important difference is that firms face a heterogeneous post-exit state in this model, while the post-exit state is assumed to be homogeneous in their model. This feature is at the core of the mechanism described in this paper.

features of a product of service, changing supply-chain systems that may incur disruptions (e.g., Hendricks and Singhal, 2005), and adopting new technologies (e.g., see Holmes, Levine, and Schmitz Jr, 2012 and the references therein).

When the entrepreneur arrives at $t = 2$, he decides whether to stay in business or cease operations after observing the realization of z_2. In the case of business exit, he switches to paid employment and earns labor income wh, in which case he enjoys the value of $\ln(wh)$. If he stays in business, he hires effective units of labor n_2 and earns a profit of $z_2^{1-\alpha} n_2^{\alpha} - wn_2$. He chooses n_2 to maximize his utility, so that the value of staying as an entrepreneur in period 2 is

$$\max_{n_2 \geq 0} \ln(z_2^{1-\alpha} n_2^{\alpha} - wn_2) = \ln(\Gamma z_2)$$

where $n_2^* = (\frac{\alpha}{w})^{\frac{1}{1-\alpha}} z_2$ and $\Gamma = (1-\alpha)(\frac{\alpha}{w})^{\frac{1}{1-\alpha}}$. Therefore, the value function at the beginning of period 2 can be summarized as

$$V_2(z_2, h) = \max\{\ln(\Gamma z_2), \ln(wh)\}$$

Note that the entrepreneur stays in business if and only if $z_2 \geq \frac{wh}{\Gamma}$. Taking $V_2(z_2, h)$ into account, the entrepreneur in period 1 chooses labor input n_1 and experiment risk Δ to maximize expected lifetime utility. Specifically, he solves the problem

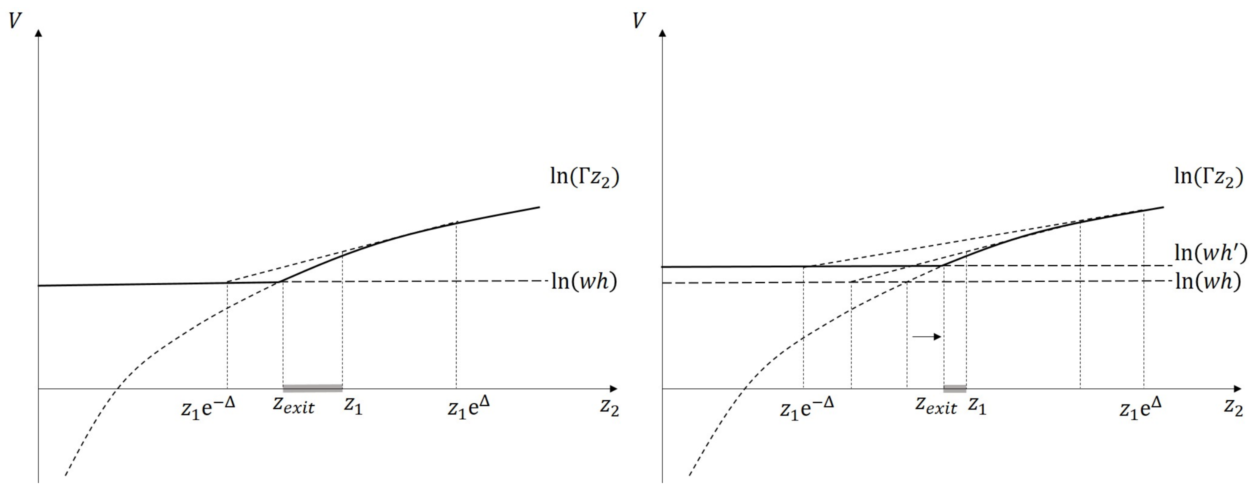
$$V_1(z_1, h) = \max_{n_1 \geq 0, \Delta \geq 0} \ln(z_1^{1-\alpha} n_1^{\alpha} - wn_1) + \beta\big\{e^{-\gamma\Delta} \cdot V_2(z_1 e^{\Delta}, h) + (1 - e^{-\gamma\Delta}) \cdot V_2(z_1 e^{-\Delta}, h)\big\}$$

where β is the time discount factor. The entrepreneur chooses n_1 to maximize period 1 profits, and thus $n_1^* = (\frac{\alpha}{w})^{\frac{1}{1-\alpha}} z_1$. The object of interest is the optimal Δ^*. It can be shown that for a given z_1, there exists $h^*(z_1)$ such that

$$\Delta^* = \begin{cases} \ln(wh) - \ln(\Gamma z_1) + \frac{1}{\gamma} & h \geq h^*(z_1) \\ \bar{\Delta}(\gamma) & 0 \leq h < h^*(z_1) \end{cases} \tag{1}$$

where $\bar{\Delta}(\gamma)$ is a decreasing function of γ. The solution is derived in Appendix A. The model predicts that an increase in h leads to a larger Δ^*, unless h is too low relative to z_1.

The core mechanism behind this result is the option value effect. The outside option of switching to paid employment provides a lower bound in the value function of the entrepreneur. This can be seen in Figure 1a, in which the value function in $t = 2$ is the upper envelope of the two occupation-specific values $\ln(wh)$ and $\ln(\Gamma z_2)$. The value function is locally convex around exit threshold z_{exit} due to the lower bound, and an entrepreneur who has z_1 around this region can increase his expected utility by introducing a risk in z_2. This endogenous risk-taking behavior is modeled by Vereshchagina and Hopenhayn (2009) in an entrepreneurship context.[8]



(a) h case

(b) h and h' comparison

Figure 1: Value function in period 2

Notes: Figure (a) shows the value functions of entrepreneurs ($\ln(\Gamma z)$) and workers ($\ln(wh)$) in period 2. Figure (b) shows that when there is an increase in labor efficiency from h to h', distance between current productivity and the exit threshold becomes shortened, and risk-taking incentive of the entrepreneur increase.

For a positive Δ, the ex post benefit in the case of success is the value gain generated by

[8] Vereshchagina and Hopenhayn (2009) generates local convexity in the entrepreneurs' value functions along the asset dimension rather than the business productivity dimension. This is done by introducing financial constraints as specified in Evans and Jovanovic (1989) and Holtz-Eakin, Joulfaian, and Rosen (1994). The core mechanism of this paper is not affected by the presence of financial constraints as long as the business productivity follows a persistent process. I introduce financial constraints later in the quantitative model in Section 4.

improving z_2 from z_1 to $z_1 e^{\Delta}$. The ex post benefit is unbounded above, and the expected marginal benefit with respect to Δ diminishes due to the concavity of the utility function and the curvature in the success probability function. In contrast, the ex post cost in the case of failure is bounded below because of the outside option. For any z_2 realizations below z_{exit}, the entrepreneur will exercise the exit option to minimize the value loss. Hence the lower bound of the ex post cost is determined by the distance between z_1 and z_{exit}, illustrated by the bold gray line in Figure 1a. As shown in Figure 1b, an increase in outside option increases the exit threshold z_{exit}, and shortens the distance between z_1 and z_{exit} (bold gray line). Therefore an increase in h lowers the expected marginal cost of choosing a large Δ, which incentivizes risk-taking behavior.[9]

The positive relationship between h and Δ for a given z_1 generates several testable implications on firm growth and survival. First, combining the optimal Δ^* and the success probability function $e^{-\gamma\Delta}$, the firm exit probability can be derived as

$$Pr(Exit) = \begin{cases} 1 - (\frac{\Gamma z_1}{wh})^{\gamma} e^{-1} & h \geq h^*(z_1) \\ 0 & 0 \leq h < h^*(z_1) \end{cases} \tag{2}$$

Therefore, the model implies that holding z_1 constant, wh and the exit probability in period 2 should be *positively* correlated. Hence, the first prediction is derived.

Prediction 1. *Controlling for z_{t-1}, entrepreneurs with higher outside options will exhibit higher firm exit rates in t.*

The second prediction is on dispersion of firm growth. Given the specified process for z, a higher Δ^* directly implies a larger dispersion in the productivity innovation. In the simple two-period setting, entrepreneurs readily exit when they fail in their risk-taking. Thus the observed innovation in z is truncated below, and the model cannot speak to outcomes concerning dispersion. With a straightforward extension to a multiperiod setup, however, it

[9] In the quantitative model, I incorporate the direct resource cost of risk-taking. While this cost affects the incentives of individuals for each h, the comparison across different h remains unaltered.

can be shown that even when entrepreneurs stay in business in the case of failure in risk-taking, their Δ_t^* are positively associated with h for a given level of z_t. The intuition is that even though the entrepreneur may not exit in the contemporaneous period after receiving an adverse outcome in z_t, lower levels of z_t increase the probability of exiting in the *future*. Thus outside option wh still affects risk-taking incentives in the same way. Therefore, the model predicts larger dispersion of firm growth for entrepreneurs with higher outside options. This prediction is confirmed in the quantitative model, in which agents are infinitely lived.

Prediction 2. *Controlling for z_{t-1}, entrepreneurs with higher outside options will exhibit larger dispersion in growth between t and $t-1$.*

The third prediction is on average firm growth rate conditioning on survival. Because taking a business risk leads to higher probability of exit in case of failure, continuing firms are more likely to consist of risk-taking winners and non-risk-takers. Given that $z_2 = z_1 e^{\Delta^*}$ for risk-taking winners, their growth rate of z and n is $e^{\Delta^*} = \frac{wh}{\Gamma z_0} e^{\frac{1}{\gamma}}$. Since entrepreneurs with higher outside options (wh) tend to take larger risks (Δ^*), they are likely to exhibit faster growth conditioning on survival.

Prediction 3. *Controlling for z_{t-1}, entrepreneurs with higher outside options will exhibit faster growth between $t-1$ and t.*

Interaction with Nonpecuniary Motives For Self-employment Empirical studies in the firm dynamics literature indicate that typical startups in the U.S. exhibit little or no growth.[10] The risk-taking mechanism developed in this paper can explain this result if many business founders have low outside options and thus take little or no risks. An alternative hypothesis was put forward by Hurst and Pugsley (2011), who attribute this pattern to the nonpecuniary benefits of self-employment. Using an occupational choice framework, Hurst and Pugsley (2016) show that individuals with strong nonpecuniary motives tend to start

[10] For example, see Hurst and Pugsley (2011) and Decker, Haltiwanger, Jarmin, and Miranda (2014).

businesses in sectors with few scale economies and exhibit no growth. The authors find empirical support for their model using the LBD.

Incorporating their argument into this model yields a unique testable prediction. Following Hurst and Pugsley (2016), the nonpecuniary benefits of entrepreneurship can be incorporated as an additive utility term $\theta > 0$ in the value function of the entrepreneur. Thus the value function in period 2 can be rewritten as

$$V_2(z_2, h; \theta) = \max\{\ln(\Gamma z_2) + \theta, \ln(wh)\}$$

The entrepreneur in period 1 then solves the problem

$$\max_{n_1 \geq 0, \Delta \geq 0} \ln(z_1^{1-\alpha} n_1^{\alpha} - wn_1) + \theta + \beta\big\{e^{-\gamma\Delta} \cdot V_2(z_1 e^{\Delta}, h; \theta) + (1 - e^{-\gamma\Delta}) \cdot V_2(z_1 e^{-\Delta}, h; \theta)\big\}$$

By solving this problem with an strategy identical to the benchmark model, the optimal Δ^* can be characterized as

$$\Delta^* = \begin{cases} \ln(wh) - \ln(\Gamma z_1) + \frac{1}{\gamma} - \theta & h \geq h^*(z_1; \theta) \\ \bar{\Delta}(\gamma) & 0 \leq h < h^*(z_1; \theta) \end{cases} \tag{3}$$

where $h^*(z_1; \theta)$ is increasing in θ. Consequently, exit probability becomes

$$Pr(Exit) = \begin{cases} 1 - (\frac{\Gamma z_1}{wh})^{\gamma} e^{\gamma\theta - 1} & h \geq h^*(z_1; \theta) \\ 0 & 0 \leq h < h^*(z_1; \theta) \end{cases} \tag{4}$$

and the growth rate of z and n conditioning on survival is obtained as

$$e^{\Delta^*} = \frac{wh}{\Gamma z_1} e^{\frac{1}{\gamma} - \theta} \tag{5}$$

Equations (4) and (5) indicate that for a given level of h and z_1, entrepreneurs with strong nonpecuniary motives take fewer risks, and thus exhibit higher survival rates, less growth dispersion, and slower growth conditioning on survival. The intuition behind this result can be understood through Figure 1a. Adding $\theta > 0$ to the entrepreneur's value function is equivalent to a parallel upward shifting of $\ln(\Gamma z_2)$, keeping $\ln(wh)$ constant. In turn, the distance between z_1 and z_{exit} widens, increasing the expected marginal cost of risk-taking. Intuitively, stronger nonpecuniary motives increase the value cost of closing the business. Therefore, the last prediction of the simple model is obtained.

Prediction 4. *Controlling for z_{t-1}, the impact of the outside options on the exit rate, growth dispersion, and average growth conditioning on survival between $t-1$ and t is mitigated for entrepreneurs with strong nonpecuniary motives for self-employment.*

Two implications of the simple model should be highlighted. First, it is critical to control for z_{t-1} to uncover the predicted patterns in the data. As shown below, the data suggest that business founders with higher h tend to enter the market with higher initial values of z.[11] It can be seen from equation (1) that when there is a positive correlation between h and z, the unconditional correlation between h and Δ^* is ambiguous. Second, the risk-taking incentives generated by the outside option are greatest around the exit margin of z, and diminish as z takes on larger values. Thus, in an environment in which startups enter the market with low levels of z relative to older incumbents, the model predicts that young firms will exhibit larger growth dispersion, higher exit rates, and faster average growth conditioning on survival than older firms. This is a well-known feature of young business dynamics in the U.S (e.g., see Haltiwanger, Jarmin, and Miranda, 2013, Decker, Haltiwanger, Jarmin, and Miranda, 2014).

[11] This is partly driven by selection, as individuals tend to enter entrepreneurship when their business ideas are worth pursuing relative to their outside option.

3 Evidence on Outside Options and Business Risk-Taking

3.1 Data and Measurement

To empirically test the predictions established in section 2, I combine two administrative databases of the U.S. Census Bureau. The first is the Longitudinal Business Database (LBD), which tracks all U.S. non-farm private establishments and firms with at least one employee since 1976. An establishment is a specific physical location where business activities occur, and all establishments under common operational control are grouped under the same firm ID.[12] The U.S. Census Bureau identifies operational control across business entities through the Economic Censuses and the Company Organization Survey. The LBD tracks business activity information on an annual basis. Data include industry, location, employment, annual payroll, birth, death, and ownership changes (if any) at the establishment level.[13]

Firm growth is measured in four dimensions in this analysis: employment, payroll, revenue, and labor productivity. Payroll and revenue are real annual values, where the CPI-U-RS and the GDP implicit price deflator, respectively, are used for nominal-to-real conversion.[14] Labor productivity is measured by real annual revenue per worker. One limitation of this labor productivity measure is that it does not account for cross-industry differences in the contribution of intermediate inputs and prices. Thus, following Haltiwanger, Jarmin, Kulick, and Miranda (2016), all regression analyses use only within-industry variation by including industry by year fixed effects. Industry is classified by the four-digit NAICS code.

The second data is the Longitudinal Employer-Household Dynamics (LEHD), which is a matched employer-employee dataset that covers 95% of private sector jobs.[15] The LEHD

[12]It is important to distinguish establishments from firms, and the Federal Employment Identification Number (EIN) from the firm ID. While most firms start with one establishment and one EIN, high-growth firms often expand to multiple establishments and occasionally obtain multiple EINs.

[13]Employment is the number of employees reported to the IRS as of the pay period that includes March 12. For more details on the LBD, see Jarmin and Miranda (2002)

[14]For detailed description of the revenue variable in the LBD, see Haltiwanger, Jarmin, Kulick, and Miranda (2016).

[15]The LEHD also covers state and local government jobs, but not the federal government jobs.

combines data from state Unemployment Insurance (UI) earnings records and the Quarterly Census of Employment and Wages (QCEW) of the Bureau of Labor Statistics. It tracks individuals at a quarterly frequency, and provides information on earnings, workplace identifiers, and demographics (e.g., age, race, gender).[16] The highest level of business unit ID in the LEHD is the federal EIN. I integrate federal EIN information from the Business Register with the LBD, and use the crosswalk developed by Haltiwanger et al. (2014) to merge the LEHD and LBD.[17]

By combining the two datasets, I construct a longitudinal sample of 1.7 million startup firms. This sample is composed of 16 cohorts of startups established between 1999 and 2014 in 31 states.[18] The data contain longitudinally stable identifiers of individuals and firms, which enables tracking of individuals' career trajectories, and the formation, growth, and dissolution of the firms they create. For sole proprietorships, founders are identified based on their income tax filings (Schedule C) and EIN applications (Form SS-4). For non-sole-proprietor firms (e.g., corporations), however, business ownership information is not available in the data. Thus, the founders of these firms are approximated with individuals who (1) appear in the initial quarter of business operation, (2) stay within the business for at least three quarters, and (3) are one of the top three earners in the second quarter of operation. This approximation method is a modified version of the approach of Kerr and Kerr (2016), which is frequently adopted in entrepreneurship studies that use the LEHD. Earnings rankings are measured in the second quarter to ensure that the individuals stayed within the firm throughout the quarter, given condition (2). In Appendix B, I show that the empirical results are broadly robust to restricting the sample only to sole proprietors.[19]

[16]For more details on the LEHD, see Abowd et al. (2009).

[17]Due to the different processing of EINs by the IRS and states, some fraction of startup EINs first appear in the two databases in different years. I only include EINs that show consistent startup timing in both datasets: I require that an EIN that belongs to a startup firm ID in the LBD in year t must appear in the LEHD either in the second, third, or fourth quarter of year $t-1$, or the first quarter of year t.

[18]The 31 states are CA, CO, FL, GA, HI, ID, IL IN, KS, LA, MD, ME, MN, MO, MT, NC, ND, NJ, NM, NV, OR, PA, RI, SC, SD, TN, TX, VA, WA, WI, and WV. Because states joined the LEHD program in a sequential manner, there is a trade-off between state coverage and time length in constructing a balanced sample.

[19]In future drafts, additional robustness checks on founder identification will be conducted.

The key object of interest in this analysis is the outside options of the business founders. In the simple model in section 2, outside options are defined by the labor income founders expect to earn in the case of firm exit. In the empirical analysis, founders' outside option is proxied by their real annual labor earnings prior to business entry.[20] This is based on the assumption that a founder's expected income after his business failure would be in a range similar to his prior earnings. While some empirical studies in the literature supports this assumption (e.g., Williams, 2000; Bruce and Schuetze, 2004), I also report a simple OLS regression of post-entrepreneurship earnings on prior earnings in Table 1, in which both objects are normalized by economy-wide average real earnings to remove the aggregate time trend. I find a statistically significant coefficient of 0.52, and that prior earnings alone account for 30% of the variation in post-entrepreneurship earnings.

Businesses are not included in the analysis if their founders did not have any prior earnings records. However, I find that average and median founder ages are around 39, and that most founders have at least one year of prior earnings record. Therefore, I conclude that such sample selection is not likely to bias the results. I further restrict the sample to the businesses where the average founder ages are between 25 and 54. The upper bound is imposed to stay reasonably far from any retirement considerations and the lower bound is set to increase the probability of capturing the young founders' full-year and full-time jobs prior to business entry. The empirical results are robust to the sample without the age restrictions.

3.2 Descriptive Statistics

Table 2 provides summary statistics of the sample. Panel A shows the employment and productivity distribution of startups and employment growth rates of year-to-year continuers up to age five. The employment growth rate between year $t-1$ and t is calculated as $\frac{Emp_t - Emp_{t-1}}{(Emp_t + Emp_{t-1})/2}$. This measure is known as the Davis-Haltiwanger-Schuh (DHS) growth rate

[20] Specifically, I measure prior earnings by the sum of real quarterly earnings from the most recent four consecutive full-quarter main jobs. The CPI-U-RS is used for nominal-to-real conversion.

Table 1: Prior earnings and Post earnings

	Log (relative) Post Earnings
Log (relative) Prior Earnings	0.520***
	(0.001)
Constant	-0.115***
	(0.001)
Obs.	1090000
R-sq	0.300

Notes: The table reports results for OLS regressions in which the independent variable is founder log prior earnings and the dependent variable is the same individual's post-entrepreneurship earnings. Observation counts are rounded to the nearest 10,000 to avoid potential unwarranted disclosure. Prior earnings are measured as the sum of real quarterly earnings at the most recent four consecutive full-quarter main jobs prior to business entry. Likewise, post earnings are measured as the sum of real quarterly earnings at the first four consecutive full-quarter jobs post business exit. Both earnings are normalized by economy-wide average real earnings obtained from the LEHD QWI. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

(Davis et al. (1996)), and is standard in the firm dynamics literature.[21] The employment growth rate distribution is weighted by employment. Weighting the distribution by employment, together with the use of the DHS growth measure, minimizes the negative relationship between size and growth generated by scale differences. Panel A in Table 2 reconfirms the previous findings in the literature. Most startup firms are small, and typical continuing young firms exhibit only little growth; the median growth rate is 1%. However, young firm growth rates show large dispersion and positive skewness, driving the mean up to 6.5%.

The relative labor productivity of each startup is measured as the deviation from its own industry's average labor productivity in the same year. Reported labor productivity statistics are calculated from an (unweighted) distribution that combines all observations between 1999 and 2013 across all industries. Labor productivity for the average startup firm is 4.9% lower than its industry's average. This estimate is in line with Foster et al. (2001), who find that entering plants in the U.S. manufacturing sector in 1987 had 7% to 8% lower

[21] The DHS growth measure mitigates the problem known as the "regression to the mean effect," and it is symmetric around zero. It is identical to the log differences up to a second-order approximation. For details, see Törnqvist et al. (1985), Davis et al. (1996), and Haltiwanger et al. (2013).

Table 2: Summary Statistics

	Obs.	Mean	Median	St. Dev.	p90	p10
Panel A. Firm Distributions						
Age 0 employment	1.7m	6.7	3	21.4	14	1
Age 0 relative labor productivity	1.7m	-.049	-.024	1.07	1.18	-1.24
Age 1-5 emp. growth (continuers)	4.4m	.065	.01	.45	.56	-.38
Panel B. Founder Distributions						
Founder log prior earnings	1.7m	10.6	10.6	0.83	11.61	9.62
Founder age	1.7m	38.9	38.7	7.48	49.5	28.7

Notes: The table reports summary statistics for the data. To avoid potential unwarranted disclosure, median, 90th, and 10th percentiles are calculated as the averages of their one-percentile neighborhood (e.g., p90 = (p89 + p91)/2). Relative labor productivity for each startup is measured by the deviation from its industry average labor productivity in its startup year, and the statistics are calculated from the (unweighted) distribution that combines observations between 1999 and 2013 across all industries. Prior earnings of each founder are measured by the sum of real quarterly earnings at four consecutive full-quarter main jobs prior to the startup quarter. Real quarterly earnings are evaluated in 2012 Q1 dollars using the CPI-U-RS. If more than one founder is identified for a given business, averages are taken across founders.

labor productivity than incumbent plants. As stated above, average and median founder ages are around 39, indicating that typical startup founders are at the peak of their prime working age. Converting logs to levels, the average founder prior earnings is around $40,134 in 2012 dollars.

Table 3 shows the relationship between founder prior earnings and their initial performance, viewed through a simple OLS regression. Age 0 employment and relative labor productivity are regressed on founder log prior earnings. Industry by year fixed effects are included for the reasons explained above. Unsurprisingly, founders with higher prior earnings tend to start businesses with more employees and higher productivity levels. Estimated coefficients indicate that a one standard deviation increase in log prior earnings is associated with 1.93 more employees and 12.5% higher labor productivity in the firm's first year of operation.

Table 3: Prior Earnings and Initial Performance

	Age 0 emp.	Age 0 relative labor productivity
Founder log prior earnings	2.337***	0.151***
	(0.343)	(0.0108)
Ind-Year FE	Yes	Yes
Obs.	1.7m	1.7m
R-sq	0.06	0.21

Notes: The table reports results for OLS regressions in which the independent variable is founder log prior earnings. Industry by year fixed effects are included in both regressions, where industry is defined by the four-digit NAICS code. Standard errors are clustered at the industry (NAICS4) level. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

3.3 Regression Analysis

3.3.1 Prediction 1: Firm Exit

Do higher outside options of business founders predict higher firm exit rates? To answer this question, I estimate linear probability regression models in which the dependent variable is the firm exit indicator.[22] Results are reported in Table 4. The first column shows the simplest case in which prior earnings is the only independent variable. The estimated coefficient indicates that the unconditional correlation between prior earnings and firm exit is negative. As explained in Section 2, however, it is critical to control for lagged business productivity to uncover the patterns predicted by the model. This is particularly important as the findings reported in Table 3 combined with Equation (1) imply an ambiguous unconditional correlation between prior earnings and exit rates.

Indeed, regression results support the model's prediction when lagged business productivity indicators are included in the regression. Column (2) shows results with lagged firm size and firm age controls, as well as industry by year fixed effects. Firm size is known to be highly correlated with firm productivity, and including it turns the coefficient on prior earnings from negative to positive. When labor productivity is also included in the regression

[22]Results are robust to using logit or probit regressions.

Table 4: Firm Exit Regressions

Dependent Variable: Firm exit indicator	(1)	(2)	(3)	(4)
Log prior earnings	-0.005**	0.003***	0.019***	0.019***
	(0.002)	(0.001)	(0.001)	(0.001)
Lagged log employment		-0.039***	-0.055***	-0.055***
		(0.001)	(0.002)	(0.002)
Lagged log labor prod.			-0.082***	-0.083***
			(0.005)	(0.005)
Lagged log wage				-0.001
				(0.003)
Founder average age				-0.001***
				(0.000)
Founder male share				0.004**
				(0.002)
Ind-Year FE	No	Yes	Yes	Yes
Firm age FE	No	Yes	Yes	Yes
State FE	No	No	No	Yes
Obs. (firm-year)	4920000	4920000	4920000	4920000
R-sq	0.00	0.03	0.09	0.10

Notes: The table reports results for a linear probability regression in which the dependent variable is firm exit indicator. Observation counts are rounded to the nearest 10,000 to avoid potential unwarranted disclosure. Standard errors are clustered at the industry (NAICS4) level. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

as in column (3), the positive relationship between prior earnings and firm exit probability is strengthened by an order of magnitude. Column (4) shows that this result is robust to including additional controls. First, I include the lagged log firm wage as an additional indicator of firm productivity, where the firm wage is calculated as the payroll per worker in the first quarter of each year. Second, I control for the average age and fraction of males among founders within each firm to account for individual characteristics that might be correlated with both prior earnings and their risk preferences. Older individuals may be more risk-averse, and males are known to be more prone to risk-taking (Laasch and Conaway, 2009). To the extent that age is positively associated with prior earnings and that gender wage gaps exist, omitting age and gender will introduce a downward and upward bias, respectively, in the estimate of the coefficient on prior earnings. Lastly, state fixed effects are included, as some states have a more dynamic business environments than others, and business dynamism is known to be positively associated with labor market conditions (Davis and Haltiwanger, 2014).

3.3.2 Prediction 2: Growth Dispersion

Do business founders with better outside options exhibit larger firm growth dispersion? To answer this question, a firm-level growth dispersion measure is constructed following Castro, Clementi, and MacDonald (2009). To begin with, I compute the portion of firm growth that is systematically predicted by firm-level and aggregate factors by estimating the following regression model:

$$\Delta \ln Y_{ijt} = \beta_0 + \beta_1 ln(emp)_{ijt-1} + Firmage_{ijt} + \eta_{jt} + \alpha_i + \epsilon_{ijt} \tag{6}$$

where $\Delta \ln Y_{ijt}$ is the DHS growth rate of either log revenue or log labor productivity, $Firmage_{ijt}$ is a series of dummies for firm age, η_{jt} are industry by year fixed effects, and α_i is a firm fixed effect. Only year-to-year continuers are included in the estimation. The object of interest is $\hat{\epsilon}_{ijt} = \Delta \ln Y_{ijt} - \widehat{\Delta \ln Y_{ijt}}$, the deviation of growth from its conditional mean. Larger *firm*-level growth dispersion corresponds to larger squared deviations from its conditional mean, $\hat{\epsilon}^2_{ijt}$.[23] It is assumed that $\epsilon^2_{ijt} = f(X_{ijt}) + \nu_{ijt}$, where X_{ijt} is a vector of factors that are systematically related to firm-level growth dispersion. Approximating $f(\cdot)$ linearly, I estimate regression equation (7) to test the model prediction in which $\tilde{X}_{ijt}$ is a vector of factors other than log prior earnings. Table 5 shows the results.

$$\epsilon^2_{ijt} = \beta_0 + \beta_1 \text{log prior earnings}_i + \beta_2 \tilde{X}_{ijt} + \nu_{ijt} \tag{7}$$

Table 5: Growth Dispersion Regressions

	$\epsilon^2(\Delta$ Rev)	$\epsilon^2(\Delta$ Rev)	$\epsilon^2(\Delta$ Prod)	$\epsilon^2(\Delta$ Prod)
Log prior earnings	0.014***	0.007***	0.009***	0.004***
	(0.000)	(0.000)	(0.000)	(0.000)
Lagged log employment		-0.003***		-0.011***
		(0.000)		(0.000)
Ind-Year FE	No	Yes	No	Yes
Firm age FE	No	Yes	No	Yes
Obs. (firm-year)	4410000	4410000	4410000	4410000
R-sq	0.00	0.03	0.00	0.02

Notes: The table reports results from estimating Equation (7). ϵ^2_{ijt} (Δ Rev) and ϵ^2_{ijt} (Δ Prod) are the squared deviations obtained from Equation (6), where Y_{ijt} are revenue and labor productivity, respectively. Observation counts are rounded to the nearest 10,000 to avoid potential unwarranted disclosure. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

Results are consistent with the model prediction: Higher log prior earnings predict larger firm-level growth dispersion. ϵ^2_{ijt} (Δ Rev) and ϵ^2_{ijt} (Δ Prod) are the squared deviations obtained from estimating Equation (6), where Y_{ijt} are revenue and labor productivity, respectively. Two sets of regression models are estimated for robustness analysis. In the first and third columns, only prior earnings is used as the independent variable. The second and fourth columns add controls for firm size, age, and industry by year fixed effects. Controlling

[23] The results are robust to using absolute deviations $|\hat{\epsilon}_{ijt}|$.

for firm age accounts for other mechanisms that can explain the high growth dispersion of young firms, such as the learning and selection effects pioneered by Jovanovic (1982). Industry by year fixed effects are included to control for time-varying industry-level factors such as uncertainty shocks (see, e.g., Bloom, 2009) that can affect firm-level idiosyncratic growth dispersion.

3.3.3 Prediction 3: Growth of Continuers

The simple model also predicts that conditioning on survival, entrepreneurs with high outside options exhibit faster firm growth. To test this prediction, firm growth is regressed on the full set of controls used in Section 3.3.1. Firm growth is measured in four dimensions: revenue, labor productivity, payroll, and employment. All growth rates are measured using the DHS method. Results are reported in Table 6.

Results are consistent with the model's prediction, with the exception of employment growth. A one standard deviation increase in log prior earnings is associated with a 1.1% annual increase in revenue, a 1.6% increase in labor productivity, a 0.8% increase in payroll, and a 0.5% decline in employment on average. Note that these are estimates only of the direct effects of prior earnings in each year. The negative average effect of log prior earnings on employment growth can be explained by the larger growth dispersion of founders with higher prior earnings. That is, founders with higher prior earnings must exhibit faster growth conditioning on expanding, and also faster decline conditioning on contracting. If the latter effect dominates the former, one may find the impact of prior earnings to be negative *on average*. To see whether this explanation is supported by the data, employment growth regressions are re-estimated separately for firms with positive employment growth (expansions) and negative employment growth (contractions). Results are shown in Table 7, in which other control variables are suppressed for simple exposition. Estimation results are consistent with the explanation, indicating that employment growth patterns are also consistent with model predictions.

Table 6: Growth Regressions for Continuers

	Δ Revenue	Δ Prod.	Δ Payroll	Δ Emp.
Log prior earnings	0.014***	0.02***	0.01***	-0.006**
	(0.002)	(0.003)	(0.002)	(0.002)
Lagged log employment	-0.013***	0.048***	0.007***	-0.063***
	(0.003)	(0.003)	(0.002)	(0.004)
Lagged log labor prod.	-0.05***	-0.168***	0.098***	0.117***
	(0.005)	(0.005)	(0.004)	(0.006)
Lagged log wage	0.014***	-0.002	-0.104***	0.019***
	(0.003)	(0.003)	(0.003)	(0.005)
Founder average age	-0.002***	-0.000**	-0.001***	-0.002***
	(0.000)	(0.000)	(0.000)	(0.000)
Founder male share	0.013***	0.032***	0.007**	-0.019***
	(0.003)	(0.005)	(0.003)	(0.005)
Ind-Year FE	Yes	Yes	Yes	Yes
Firm age FE	Yes	Yes	Yes	Yes
State FE	Yes	Yes	Yes	Yes
Obs.	4410000	4410000	4410000	4410000
R-sq	0.05	0.11	0.06	0.12

Notes: The table reports results for OLS regression of firm growth on prior earnings. All growth measures are calculated as the DHS growth rate. Observation counts are rounded to the nearest 10,000 to avoid potential unwarranted disclosure. Standard errors are clustered at the industry (NAICS4) level. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

Table 7: Conditional Employment Growth Regressions

	Δ Emp > 0	Δ Emp < 0
Log prior earnings	0.016***	-0.023***
	(0.002)	(0.002)
Lagged log labor prod.	0.06***	0.049***
	(0.004)	(0.002)
Lagged log employment	-0.164***	0.095***
	(0.003)	(0.003)
Ind-Year FE	Yes	Yes
Firm age FE	Yes	Yes
Full controls	Yes	Yes
Obs.	1520000	1190000
R-sq	0.38	0.17

Notes: The table reports results for the OLS regression in which the dependent variable is annual employment growth rate, conditioning on employment expansions and contractions separately. Observation counts are rounded to the nearest 10,000 to avoid potential unwarranted disclosure. Standard errors are clustered at the industry (NAICS4) level. ***, **, and * indicates significance at the 1%, 5%, and 10% levels, respectively.

3.3.4 Prediction 4: Interaction with the Hurst-Pugsley Small Business Sector

Lastly, the model predicts that all results presented so far will be mitigated for business founders with a strong preference for self-employment. Although this preference is not observable directly, Hurst and Pugsley (2016) show that individuals with strong nonpecuniary motives are likely to be concentrated in sectors with small natural scale. Their intuition is that if the primary goal is to become a business owner and not to earn large profits, those individuals will do so in the most cost-effective way. In their model, differences in natural scale are driven by heterogeneous fixed costs of operation; hence, small natural scale sectors are the least costly to enter. Hereafter, such small natural scale sectors are labeled collectively as the Hurst-Pugsley (HP) sector.

Following Hurst and Pugsley (2016), the HP sector is defined by the top 40 (out of 294) four-digit NAICS industries in terms of small business intensity. Small business intensity of industry j, x_j, is calculated by

$$x_j = \frac{s_j}{\sum_k s_k}$$

where s_j is the number of small businesses (fewer than 20 employees) in industry j. The denominator is the sum of s_k across all industries. Then an indicator variable HP_j is created, which takes a value of one if industry j is in the HP sector and zero otherwise. The regression models presented above are re-estimated including the interaction term between the HP indicator and log prior earnings. Results for key dependent variables are presented in Table 8. For all regression models, the estimated coefficients on HP interaction terms have the opposite signs as the coefficients on log prior earnings, and in many cases are statistically significant. Therefore, the results are consistent with the model prediction.

Table 8: Regression Results with the Hurst-Pugsley Sector Interactions

	(1) Exit	(2) Δ Rev	(3) Δ Prob	(4) Δ Emp > 0	(5) Δ Emp < 0	(6) ϵ^2 (Δ Prod)
Log prior earnings	0.023***	0.022***	0.027***	0.018***	-0.027***	0.008***
	(0.002)	(0.002)	(0.003)	(0.003)	(0.002)	(0.000)
HP × log prior earnings	-0.006***	-0.011***	-0.01**	-0.003	0.006	-0.002***
	(0.001)	(0.003)	(0.005)	(0.005)	(0.004)	(0.000)
Lagged log labor prod.	-0.083***	-0.059***	-0.168***	0.06***	0.049***	
	(0.005)	(0.005)	(0.005)	(0.004)	(0.002)	
Lagged log employment	-0.055***	-0.013***	0.047***	-0.164***	0.095***	-0.003***
	(0.002)	(0.003)	(0.003)	(0.003)	(0.003)	(0.000)
Ind-Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Firm age FE	Yes	Yes	Yes	Yes	Yes	Yes
Full controls	Yes	Yes	Yes	Yes	Yes	No
Obs.	4920000	4410000	4410000	1520000	1190000	4410000
R-sq	0.10	0.05	0.11	0.38	0.17	0.03

Notes: The table reports results for linear regressions re-estimated after including the interaction between HP indicator and log prior earnings. Observation counts are rounded to the nearest 10,000 to avoid potential unwarranted disclosure. Standard errors are clustered at the industry level (NAICS4). ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

The Need for a Quantitative General Equilibrium Model The empirical evidence mostly supports the prediction from the simple model that if business founders have better outside options, they tend to take larger business risks and thus exhibit a more up-or-out type of firm dynamics. This result suggests that factors affecting the outside options of entrepreneurship (e.g. labor market frictions) would alter risk-taking behavior by startups and growth dynamics along their life cycle. However, whether this channel can be translated into a quantitatively significant effect on the aggregate economy is unclear. For instance, because the share of business activity accounted for by young firms is small, changes in their risk-taking behavior may not generate sufficiently strong forces to affect macro-level outcomes.

Therefore, one needs a structural macroeconomic model disciplined by data to investigate the quantitative importance of outside options and young firms' risk-taking behavior in shaping aggregate outcomes. Structural macro models also provide a useful laboratory to conduct experiments that cannot be done otherwise, such as altering the outside options of startup entrepreneurs while holding all other factors constant. In the following section, I embed the simple model presented in Section 2 into a heterogeneous agent general equilibrium model to conduct this analysis.

4 General Equilibrium Model

4.1 Model Description

Environment There is a continuum of individuals and time is discrete. At the beginning of each period t, individuals randomly die with probability ζ, and the same mass of individuals newly enter the economy. Individuals live infinitely unless they are hit by the death shock. Upon entry to the economy, individuals receive assets a_0 from a distribution $\mu_a(a)$, and draw their effective units of labor h from a Pareto distribution $F(h)$,

$$F(h) = 1 - h^{-\lambda}, \qquad h \geq 1 \tag{8}$$

Also, individuals draw their initial business productivity z from a log-normal distribution where

$$\ln(z) \sim \mathcal{N}(\mu_z(h), \sigma_z^2) \tag{9}$$

It is assumed that

$$\mu_z(h) = \underline{z} + \rho h \tag{10}$$

where $\underline{z}$ is a parameter that governs the overall location of the z distribution. The dependence of $\mu_z(h)$ on h reflects the notion that individuals with different levels of effective units of labor may have access to different types of business ideas. Individuals also draw preference for entrepreneurship $\theta \in \{0, \bar{\theta}\}$, where $\theta = \bar{\theta} > 0$ with probability p_θ and 0 with probability $1 - p_\theta$. For simplicity, θ is assumed to stay constant throughout individuals' lifetime.

Occupational Choice At the beginning of each period, individuals decide whether to become a worker or an entrepreneur. When individuals choose to become paid workers, they supply their effective units of labor h to entrepreneurs and earn wages wh, where w is the wage rate per effective unit of labor. Workers receive interest payments by depositing their assets a, and consume c out of $(1 + r)a + wh$. Thus, the workers' problem is defined by

$$\begin{aligned} V^W(a, z, h, \theta) &= \max_{c \geq 0} \ u(c) + \beta(1 - \zeta)V(a', z, h, \theta) \\ \text{s.t} \ \ a' &= (1 + r)a + wh - c \geq 0 \end{aligned} \tag{11}$$

For simplicity, h is assumed to stay constant if the individuals stay in paid employment. Hereafter, next-period variables are denoted with superscript $'$. The value function V is defined as

$$V(a, z, h, \theta) = \max\{V^W(a, z, h, \theta), V^E(a - \phi, z, h, \theta)\} \tag{12}$$

where $V^E(a, z, h, \theta)$ is the value of becoming an entrepreneur and ϕ is a fixed entry and exit cost. When individuals enter entrepreneurship, they employ effective units of labor n

and rent physical capital k to produce output y via production function $y = z^{1-\nu}(k^{\alpha}n^{1-\alpha})^{\nu}$. $\alpha \in (0,1)$ is the capital production share, and $\nu \in (0,1)$ is the decreasing returns to scale parameter, which stems from the limited span of control as in Lucas (1978).

Risky Experimentation and Incremental Innovation A key feature of the model is that entrepreneurs can attempt to enhance their productivity levels through two means of innovation: risky experimentation or incremental innovation. When an entrepreneur engages in risky experimentation, his next-period business productivity z' evolves according to a binomial risky process, as in the simple model in Section 2:

$$z' = \begin{cases} ze^{\Delta^R} & \text{w/ prob.} \quad e^{-\gamma\Delta^R} \\ ze^{-\Delta^R} & \text{w/ prob.} \quad 1 - e^{-\gamma\Delta^R} \end{cases}$$

That is, his business productivity either increases or decreases by Δ^R percent. $\gamma > 0$ is the success probability elasticity of risk choice, and Δ^R is assumed to be a choice variable implying that entrepreneurs can decide how much risk to take. To conduct risky experiment, entrepreneurs must pay an experimentation cost $Fz > 0$.

When an entrepreneur attempts to achieve incremental innovation, his next-period business productivity z' evolves as

$$z' = \begin{cases} ze^{\Delta^I} & \text{w/ prob.} \quad u \\ z & \text{w/ prob.} \quad 1 - u \end{cases}$$

where the ex post outcome is bounded below by the status quo. Innovation step size $\Delta^I > 0$ is a fixed parameter, and success probability u is a choice variable. Entrepreneurs can increase u subject to the cost function

$$R_I = \chi z u^{\psi}$$

where $\frac{1}{\psi} < 1$ is the success probability elasticity of research cost R_I, and $\chi > 0$ is a scaling parameter. This is a standard functional specification for R&D activity in the endogenous growth and innovation literature.[24]

Incremental innovation is introduced in the quantitative model to avoid potential bias in quantitative evaluation of the risky experimentation channel. In the absence of incremental innovation, the firm productivity distribution in the model is completely pinned down by the initial z distribution and the risky experimentation performed by young firms whose z's are not too far from the exit margin. In reality, however, large, mature firms often engage in innovation activities, including R&D. Therefore, the quantitative relevance of the risky experimentation channel will be overemphasized if aggregate research statistics are targeted in calibration and the innovation tools used by mature incumbent firms are not introduced in the model.

Financial Market There are financial intermediaries who own a technology that transforms consumption goods into physical capital, and vice versa, at a one-to-one rate. Financial intermediaries receive deposits from workers and entrepreneurs, and use the transformation technology to rent physical capital to entrepreneurs. It is assumed that the financial market is perfectly competitive, and thus the capital rental rate is $r + \delta$, where $\delta > 0$ is the capital depreciation rate. Financial intermediaries require collateral from entrepreneurs when engaging in capital rental contracts. Denoting $m \geq 0$ as the collateral amount, entrepreneurs can borrow capital only up to a multiple of their collateral, i.e., $k \leq \lambda m$.[25] At the beginning of each period, entrepreneurs decide how much to consume out of their assets a, and the remainder $m = a - c$ is put up as collateral. Financial intermediaries pay interest on the collateral at the beginning of the following period.

[24] For example, see Acemoglu, Akcigit, Bloom, and Kerr (2013) and Akcigit and Kerr (forthcoming).

[25] This simple financial constraint is widely used in this class of models due to its tractability. For example, see Buera and Shin (2013) and Moll (2014).

Entrepreneurs' Optimization Problem The optimization problem for an entrepreneur who engages in risky experimentation can be expressed as

$$V^{E,R}(a,z,h,\theta) = \max_{c,n,k,\Delta^R} u(c)+\theta+\beta(1-\zeta)\Big[e^{-\gamma\Delta^R}\tilde{V}(a', ze^{\Delta^R}, h, \theta)+(1-e^{-\gamma\Delta^R})\tilde{V}(a', e^{-\Delta^R}, h, \theta)\Big] \tag{13}$$

$$\begin{aligned} \text{s.t } & a' = m + z^{1-\nu}(k^\alpha n^{1-\alpha})^\nu - wn - (r+\delta)k - Fz \cdot \mathbb{I}(\Delta^R > 0) \geq 0 \\ & m = (1+r)a - c \geq 0 \\ & k \leq \lambda m \end{aligned}$$

where

$$\tilde{V}(a', z', h, \theta) = \max\left\{V^E(a', z', (1-\delta_h)h, \theta),\ V^W(a'-\phi, z', h, \theta)\right\} \tag{14}$$

and $\mathbb{I}(\Delta^R > 0)$ is an indicator that takes a value of one if $\Delta^R > 0$. I assume that labor efficiency depreciates by δ_h percent if an individual stays in entrepreneurship. This assumption reflects the idea that individuals who enter entrepreneurship and then return to paid employment generally end up on a worse career trajectory.[26]

Likewise, the optimization problem for entrepreneurs who attempt to achieve incremental innovation can be expressed as

$$V^{E,I}(a,z,h,\theta) = \max_{c,n,k,u} u(c) + \theta + \beta(1-\zeta)\Big[u\tilde{V}(a', ze^{\Delta^I}, h, \theta) + (1-u)\tilde{V}(a', z, h, \theta)\Big] \tag{15}$$

$$\begin{aligned} \text{s.t } & a' = m + z^{1-\nu}(k^\alpha n^{1-\alpha})^\nu - wn - (r+\delta)k - \chi z u^\psi \geq 0 \\ & m = (1+r)a - c \geq 0 \\ & k \leq \lambda m \end{aligned}$$

Entrepreneurs in a given period decide between risky experimentation or incremental innovation (or no innovation), and cannot conduct both. Hence, the value function of being an

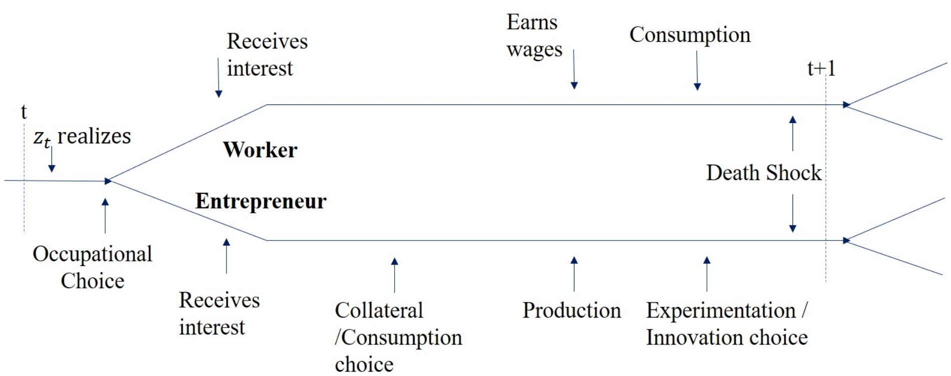
[26] For example, see Williams (2000), Bruce and Schuetze (2004), and Baptista, Lima, and Preto (2012).

entrepreneur is

$$V^E(a, z, h, \theta) = \max\{V^{E,R}(a, z, h, \theta), V^{E,I}(a, z, h, \theta)\} \tag{16}$$

Figure 2 summarizes the timing of events.

Figure 2: Timing of Events



4.2 Stationary Recursive Competitive Equilibrium

In the quantitative analysis, I focus on a stationary recursive competitive equilibrium. The interest rate r is treated as exogenously given, so that the model can be considered to be a small open economy. The state variables of individuals are assets, effective labor, business productivity, preference for entrepreneurship, and occupation. $\boldsymbol{A} = [0, \infty)$ is the set of possible asset holdings a, and $\boldsymbol{Z} = [0, \infty)$ is the space of business productivity z.[27] Effective labor h is defined over $\boldsymbol{H} = [0, \infty)$, and preference for entrepreneurship θ is defined over $\boldsymbol{\Theta} = \{0, \bar{\theta}\}$ as above. Occupation is defined as $o \in \boldsymbol{O} = \{w, er, ei\}$, where each element in $\boldsymbol{O}$ represents being a worker, an entrepreneur conducting risky experimentation, and an entrepreneur conducting incremental innovation, respectively. Then the distribution of

[27]The domain of the distribution needs to be a compact set when solving the model computationally. Hence, finite upper bounds are imposed on $\boldsymbol{A}$ and $\boldsymbol{Z}$, and their values are set such that there is no mass on those points under the stationary distribution.

individuals μ is defined as a probability measure $\mu(a,z,h,\theta,o): B \to [0,1]$, where B is the Borel σ-algebra generated by the open sets of the product space $\boldsymbol{A} \times \boldsymbol{Z} \times \boldsymbol{H} \times \boldsymbol{\Theta} \times \boldsymbol{O}$. Two additional auxiliary objects are defined. First, occupational choice $o'(a,z,h,\theta,o)$: $\boldsymbol{A} \times \boldsymbol{Z} \times \boldsymbol{H} \times \boldsymbol{\Theta} \times \boldsymbol{O} \to \boldsymbol{O}$ is defined as a function that solves (12) if $o = w$, and solves (14) and (16) if $o \in \{er, ei\}$. Second, the state vector for each occupation o is defined as $\Omega_o = (a,z,h,\theta,o)$.

It is assumed that new entrants to the economy begin as workers, and randomly draw assets a from the asset distribution in the previous period. Then the distribution μ follows the law of motion $\mu' = \Phi(\mu)$, where

$$\begin{aligned}
\mu'(\tilde{a},\tilde{z},h,\theta,\tilde{o}) =&(1-\zeta)\cdot\Bigg\{\int\int_{\mathbb{1}\{\tilde{a}=a'(\Omega_w),\tilde{z}=z,\tilde{o}=o'(\Omega_w)\}}\mu(\Omega_w)dadz \\
&+ e^{-\Delta^R(\Omega_{er})}\int\int_{\mathbb{1}\{\tilde{a}=a'(\Omega_{er}),\tilde{z}=ze^{\Delta^R(\Omega_{er})},\tilde{o}=o'(\Omega_{er})\}}\mu(\Omega_{er})dadz \\
&+ (1-e^{-\Delta^R(\Omega_{er})})\int\int_{\mathbb{1}\{\tilde{a}=a'(\Omega_{er}),\tilde{z}=ze^{-\Delta^R(\Omega_{er})},\tilde{o}=o'(\Omega_{er})\}}\mu(\Omega_{er})dadz \\
&+ u(\Omega_{ei})\int\int_{\mathbb{1}\{\tilde{a}=a'(\Omega_{ei}),\tilde{z}=ze^{\Delta^I(\Omega_{ei})},\tilde{o}=o'(\Omega_{ei})\}}\mu(\Omega_{ei})dadz \\
&+ (1-u(\Omega_{ei}))\int\int_{\mathbb{1}\{\tilde{a}=a'(\Omega_{ei}),\tilde{z}=z,\tilde{o}=o'(\Omega_{ei})\}}\mu(\Omega_{ei})dadz\Bigg\} \\
&+ \zeta\cdot\int_{\mathbb{1}(o=w)} d\mu_a(a)dF(h)dG(z|h)
\end{aligned}$$

where

$$\mu_a(a) = \int\int\sum_{\theta\in\{0,\bar{\theta}\}}\sum_{o}\mu(\Omega_o)dzdh$$

A stationary recursive competitive equilibrium is defined as follows.

Definition For a given interest rate r, a *stationary recursive competitive equilibrium* is a set of value functions $\{V^W, V^{E,R}, V^{E,I}\}$, policy functions $\{a', c, k, n, \Delta^R, u, o\}$, wage rate w, and distribution μ^* such that

1. Individuals optimize:

   $V^W, V^{E,R}, V^{E,I}$ satisfy (11), (12), (13), (14), (15), and (16). Associated policy functions are as follows. $a' : \boldsymbol{A} \times \boldsymbol{Z} \times \boldsymbol{H} \times \boldsymbol{\Theta} \times \boldsymbol{O} \to \boldsymbol{A}$ is the savings decision; $c : \boldsymbol{A} \times \boldsymbol{Z} \times \boldsymbol{H} \times \boldsymbol{\Theta} \times \boldsymbol{O} \to \mathbb{R}^{++}$ is consumption; $k : \boldsymbol{A} \times \boldsymbol{Z} \times \boldsymbol{H} \times \boldsymbol{\Theta} \times \{er, ei\} \to \mathbb{R}^{+}$ is capital demand; $n : \boldsymbol{A} \times \boldsymbol{Z} \times \boldsymbol{H} \times \boldsymbol{\Theta} \times \{er, ei\} \to \mathbb{R}^{+}$ is labor demand; $\Delta^R : \boldsymbol{A} \times \boldsymbol{Z} \times \boldsymbol{H} \times \boldsymbol{\Theta} \times \{er, ei\} \to \mathbb{R}^{+}$ is the risky experimentation choice; $u : \boldsymbol{A} \times \boldsymbol{Z} \times \boldsymbol{H} \times \boldsymbol{\Theta} \times \{er, ei\} \to \mathbb{R}^{+}$ is the incremental innovation choice; and $o : \boldsymbol{A} \times \boldsymbol{Z} \times \boldsymbol{H} \times \boldsymbol{\Theta} \times \boldsymbol{O} \to \boldsymbol{O}$ is the occupational choice.

2. The labor market clears:

$$\sum_{\theta \in \{0, \bar{\theta}\}} \int n(\Omega_{er})\mu(\Omega_{er})dadzdh + \sum_{\theta \in \{0, \bar{\theta}\}} \int n(\Omega_{ei})\mu(\Omega_{ei})dadzdh = \sum_{\theta \in \{0, \bar{\theta}\}} \int \int \int h\mu(\Omega_w)dadzdh$$

3. Distribution is time-invariant:

$$\mu^*(a, z, h, \theta, o) = \Phi(\mu^*(a, z, h, \theta, o))$$

Luttmer (2012) shows that in a similar environment in which firm productivity processes follow a standard Brownian motion, a stationary distribution μ^* exists as long as the death rate ζ is large enough and the dispersion of entrants' productivity distribution is not too large. The intuition is that even though each cohort of innovating entrepreneurs moves upwards in productivity space $\boldsymbol{Z}$, their mass is reduced by rate ζ every period and eventually converges to zero. At the same time, ζ mass of individuals enter the economy at the lower part of the productivity distribution, balancing out overall growth in z achieved by entrepreneurs in the previous period. Exogenous churning induced by ζ and entailed firm entry and exit costs prevent assets from diverging.

5 Quantitative Analysis

5.1 Calibration

I calibrate model parameters to match certain key features of the U.S. non-farm private sector between 1999 and 2014. This period is chosen so that I can exploit the regression results obtained in Section 3 to discipline the model. A subset of parameters are fixed at values commonly used in the macroeconomics literature. The remaining parameters are chosen to minimize the distance between a set of equilibrium moments obtained from model simulation and their data counterparts. The model parameters are summarized in Table 9 and Table 10.

Externally Calibrated Parameters The model period is equivalent to one year. I set the time discount factor β to 0.968. The relative risk-aversion coefficient σ is set to 2, which is standard in the literature. I set the decreasing returns to scale parameter ν to 0.85, as in Atkeson and Kehoe (2007) and Midrigan and Xu (2014). I set the capital depreciation rate δ to 0.065 and the interest rate to 0.03. The R&D cost elasticity parameter ψ is set at 2, which is standard in the endogenous growth and innovation literature (e.g., see Acemoglu, Akcigit, Bloom, and Kerr, 2013 and Akcigit and Kerr, forthcoming). Labor efficiency depreciation rate of entrepreneurs, δ_h, is set at 0.03. This value is taken from existing empirical studies which find that returns to entrepreneurship experience are on average lower than the returns to experience as paid workers.[28] 3% is within the range of estimates reported in the literature. I set the labor efficiency distribution dispersion parameter η_h to 1.41, to match the 90th to 10th percentile ratio of the weekly earnings distribution reported by the BLS. The ratio is calculated as the average value between 1999 and 2014.[29]

[28]For example, see Bruce and Schuetze (2004), Kaiser and Malchow-Møller (2011), and Baptista, Lima, and Preto, 2012.

[29]The BLS computes the weekly earnings distribution percentiles from the Current Population Survey (CPS) sample that comprises of wages and salary workers who are 25 years or older.

Table 9: Externally Calibrated Parameters

Parameter		**Value**	**Source**
Discount factor	β	0.968	Standard
Risk-aversion coefficient	σ	2	Standard
Capital production share	α	0.330	Standard
Returns to scale parameter	ν	0.850	Standard
Capital depreciation rate	δ	0.065	Standard
Interest rate	r	0.030	Standard
R&D cost elasticity	ψ	2	Akcigit and Kerr (2017)
Human capital depreciation rate	δ_h	0.030	Literature
Labor efficiency dispersion	η_h	1.410	p90/p10 weekly earnings 1999-2014 (BLS)

Internally Calibrated Parameters The exogenous death rate ζ is set to 0.05 to reproduce the exit rate of mature firms (age 6 or higher) observed in the data, where the data moment is calculated as the average value between 1999 and 2014 from the Census Bureau's Business Dynamics Statistics (BDS). The implied effective discount rate $\beta(1-\zeta)$ is 0.92, which is consistent with the choice of Buera, Kaboski, and Shin (2011) and Midrigan and Xu (2014). The financial frictions parameter, λ, is set to 4.5 to match the average value of $\frac{k-m}{k}$ of all firms in the model to the average ratio of liabilities to nonfinancial assets for the U.S. nonfinancial business sector between 1999 and 2014.[30]

Hurst and Pugsley (2011) document that about 50% of startup entrepreneurs in the Panel Study of Entrepreneurial Dynamics (PSED) report nonpecuniary motives as one of the primary reasons for starting their businesses, and the majority of those entrepreneurs'

[30] I follow Buera and Nicolini (2017) and compute the statistics from the U.S. flow of funds. I measure liabilities as the sum of total liabilities of noncorporate (FL114190005.Q) and corporate (FL104190005.Q) firms in the nonfinancial sector minus the U.S. real estate owned by foreigners (FL115114005.Q) and the foreign direct investment in the U.S. (FL103192005.Q). Similarly, I measure nonfinancial assets as the sum of nonfinancial assets of noncorporate (FL112010005.Q) and corporate (FL102010005.Q) firms in the nonfinancial sector minus the U.S. real estate owned by foreigners (FL115114005.Q) and the foreign direct investment in the U.S. (FL103192005.Q).

firms remained small throughout the sample period of their study. In the model, higher values of θ induces individuals to enter entrepreneurship with smaller sizes and to take smaller risks as shown in Prediction 4 in Section 2. Thus, I jointly calibrate θ and p_θ to replicate that startup entrepreneurs with $\theta = \bar{\theta}$ comprise 50% of all startup entrepreneurs in the model, and to target the share of small firms (less than 20 employees) in the economy.

The initial distribution of z is governed by three parameters: $\underline{z}$, σ_z, and ρ. I calibrate $\underline{z}$ and σ_z to match the (employment-weighted) firm entry rate and the ratio between the average employment of entrants relative to the average employment of incumbents, respectively. To discipline ρ, I use the micro-level relationship between startup employment and log prior earnings reported in Table 3. I normalize startup employment by the average employment over all firms calculated from the BDS, which is 22.8, and obtain a normalized coefficient of 0.105. I simulate the model and create a cohort of 100,000 startup firms, and run an identical regression with the simulated data to calibrate ρ, in which employment levels of startups in the simulated data are normalized by average employment over all firms in the model.

To calibrate the elasticity of the innovation success probability with respect to risk choice, γ, I run a regression with the simulated data that is counterpart to the regression of exit on log prior earnings and firm characteristics reported in Table 4. Because the model describes a single good economy with no price heterogeneity and adjustment frictions, the simulated data do not show variation in revenue labor productivity, as in the empirical data. Therefore, I use z in the simulated data regression. I target the coefficient on log prior earnings of 0.019 reported in columns (3) and (4) of Table 4. I set the experimentation cost parameter F, another factor that strongly governs the risk-taking incentives of young firms, to match the average employment growth rate of young firms. Because the exit rates of old firms are determined by the exogenous death rate ζ, the fixed firm entry and exit cost ϕ mostly governs the exit rate of young firms. Hence, I calibrate ϕ to match the average exit rate of young firms. In the model, most incremental innovation is conducted by old firms whose z's have moved sufficiently far away from their exit margin. Therefore, I target the employment growth of old firms (age 10+) to calibrate the incremental innovation step size Δ^I. I choose the incremental innovation research cost scale parameter, χ, to match the R&D intensity of innovating firms documented by Akcigit and Kerr (forthcoming).

Table 10: Internally Calibrated Parameters

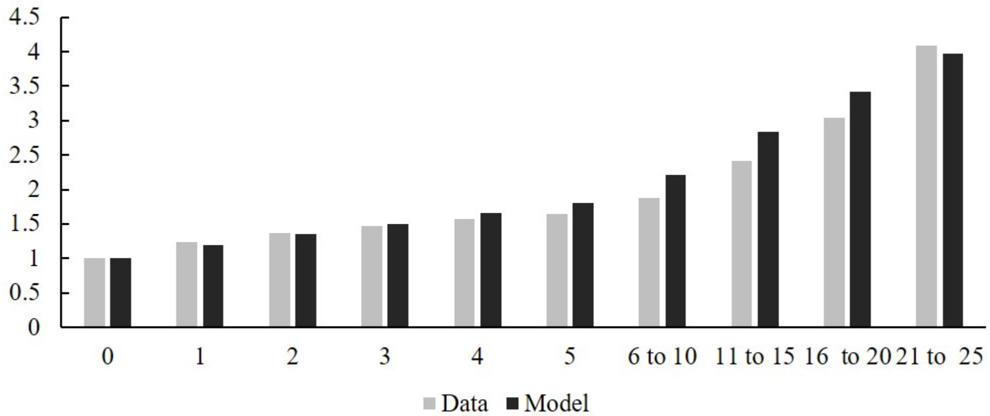
Parameter		**Value**	**Target**	**Source**	**Model**	**Data**
Financial friction	λ	4.5	Liability to nonfinancial assets	U.S. Flow of Funds	0.707	0.695
Death probability	ζ	0.05	Age 6+ exit rate	BDS 1999-2014	0.053	0.059
z distribution location	$\underline{z}$	1.0	Firm entry rate	BDS 1999-2014	0.021	0.025
z distribution dispersion	σ_z	0.6	Emp.ratio entrants to incumbents	Startup Data	0.331	0.270
z and h dependence	ρ	0.09	Reg. age 0 emp. vs. $\ln(PrEarn)$	Startup Data	0.097	0.102
Risk prob. elasticity	γ	1.2	Reg. exit. vs. $\ln(PrEarn)$	Startup Data	0.029	0.019
Risky experiment cost	F	0.16	Age 1-5 avg emp. growth	Startup Data	0.089	0.065
Firm entry / exit cost	ϕ	0.1	Age 1-5 exit rate	Startup Data	0.104	0.134
Increm. innov. step size	Δ^I	0.3	Age 10+ avg emp. growth	Acemoglu et al (2013)	0.034	0.015
Increm. innov. cost scale	χ	3.5	R&D - Sales ratio	Akcigit & Kerr (2017)	0.010	0.042
Preference for entrep.	$\bar{\theta}$	0.8	Small (emp $\leq$ 20) firm share	BDS 1999-2014	0.700	0.890
Prob($\theta = \bar{\theta}$)	p_θ	0.11	Frac. nonpecuniary entrep.	Hurst & Pugsley (2011)	0.498	0.500

5.2 Calibrated Model Properties

In this section, I document the key properties of the calibrated model. Figure 3 plots the average employment of each firm age group computed from the BDS and those calculated using the data obtained from model simulation. The BDS statistics are calculated as the average values between 1999 and 2014. Since employment in the model is expressed in the effective unit of labor, employment levels in the BDS and the model are not directly comparable. Therefore, I normalize the entrants' average employment in both data series to one and compare the slope over the life cycle. As shown in the figure, the model does a reasonable job in tracking the average size by age observed in the empirical data.

Figure 4 shows the mean employment growth rate of the continuing firms and exit rates over firm age in the model-simulated data. Both series exhibit a convex decreasing shape:

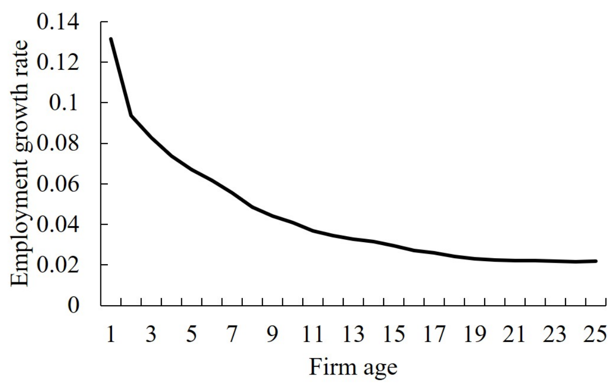
Figure 3: Average Employment by Firm Age



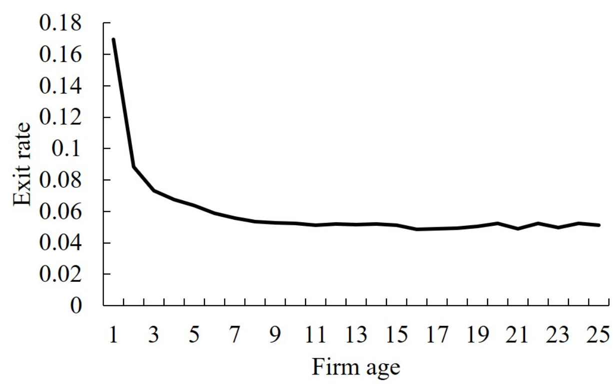
Notes: The data corresponds to the average employment by firm age from the Business Dynamics Statistics. The data values are computed as the average values between 1999 and 2014. Model statistics are calculated from a simulated data that contains a cohort of 100,000 startup firms.

younger firms exit at a higher rate, but conditioning on survival, they grow faster. This up-or-out growth dynamics of young firms implied by the model is consistent with the empirical findings in the literature (e.g., see Evans, 1987; Dunne, Roberts, and Samuelson, 1989; Haltiwanger, Jarmin, and Miranda, 2013).

Figure 4: Life-Cycle Growth and Survival Dynamics in the Model



(a) Mean Employment Growth Rate (Continuers)



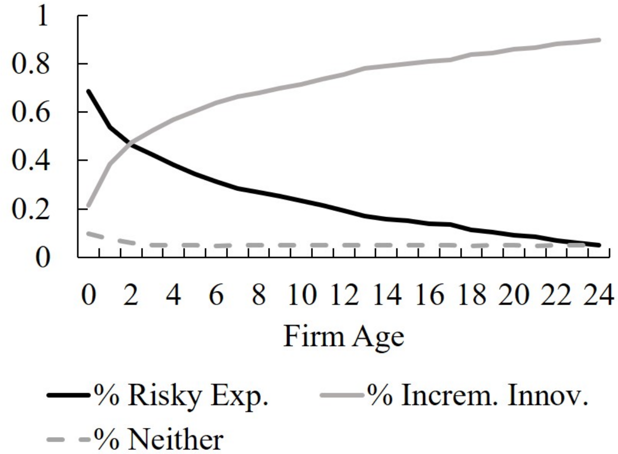
(b) Exit Rate

In this model, the decline in exit and employment growth rates with respect to firm age is driven by the higher intensity of risk-taking behavior by young firms. Young firms tend to take larger risks because of the following reasons: First, entrepreneurs start their firms with lower levels of productivity compared to those of the incumbents, and secondly, labor market clearing condition determines the wage such that startup entrepreneurs are close to their occupation switching margin. Therefore, returning to paid employment is a viable exit option for startup entrepreneurs in the case of failure from risky experimentation, which incentives them to take larger risks as shown in Section 2. As firms get older, risk-taking winners achieve an increase in their business productivity and risk-taking losers either shrink or exit. Therefore, the productivity levels of continuing firms gradually move away from the exit threshold and the entrepreneurs conduct less risky experimentation as their firms age.

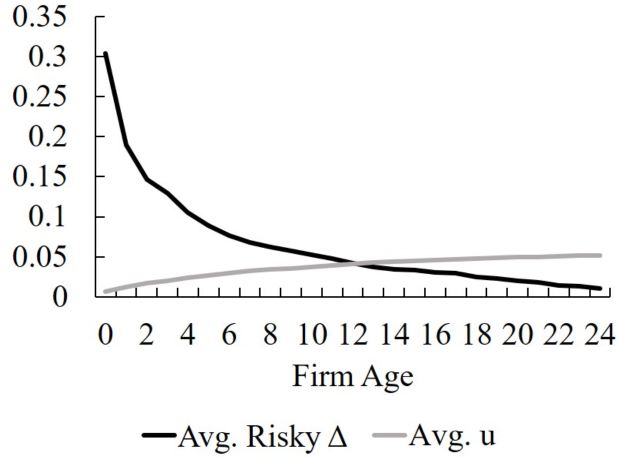
In Figure 5, I plot the experimentation and innovation patterns of continuing firms over their life cycle. Figure 5a shows the fraction of entrepreneurs who engage in risky experimentation, or incremental innovation, or neither. Figure 5b shows the average choice of Δ^R conditioning on $\Delta^R > 0$, and the average incremental innovation success probability u conditioning on $u > 0$. The fraction of entrepreneurs conducting risky experimentation and their risk-taking intensity declines as their firms age. Simultaneously, entrepreneurs gradually switch over to incremental innovation. Also in the early phase of the firm life cycle, a significant fraction of entrepreneurs do not engage in any innovation activities. For instance, at age zero, when the risk-taking incentives are the greatest, about 10% of entrepreneurs do not conduct any experimentation or innovation and 20% of entrepreneurs exert only negligible effort in incremental innovation (average u of 0.006). Therefore, about in total 30% of firms show little or no growth in productivity at age one.

A higher intensity of risky experimentation results in a more rapid pace of selection and reallocation, which in turn drives up the average productivity of the continuing firms. Since the intensity of risky experimentation is higher for younger firms, growth in average productivity declines in firm age as illustrated in Figure 6. This model implication is consistent with the recent empirical findings of Alon, Berger, Dent, and Pugsley (2017) where they show in the U.S nonfarm business sector, the relationship between firm age and productivity growth

Figure 5: Experimentation and Innovation of Continuing Firms



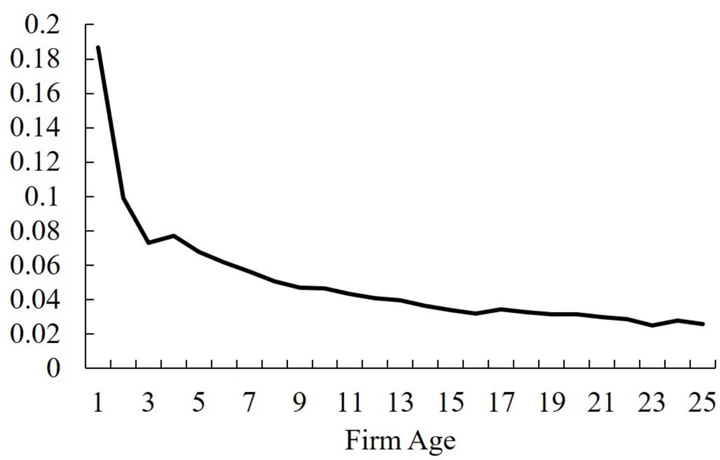
(a) Share of Risky Exp. vs. Incremental Innov.



(b) Experimentation / Innovation Intensity

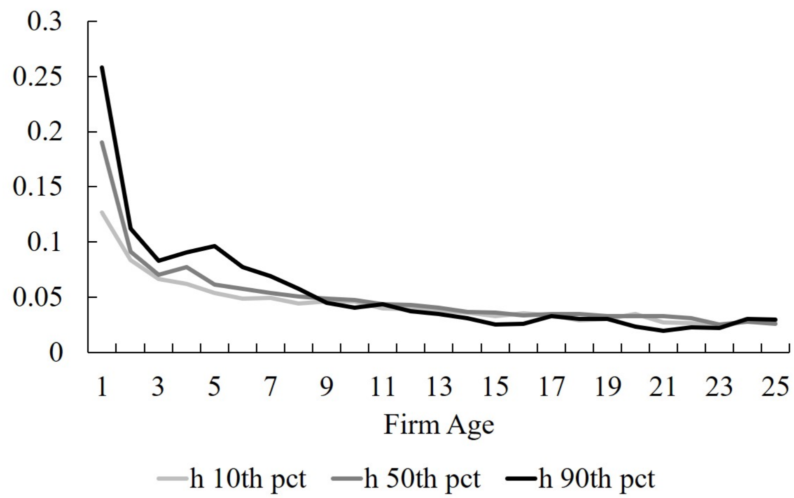
is downward sloping and convex, and that most of the productivity growth is concentrated among firms less than five years old.

Figure 6: Growth of Average Productivity by Firm Age



In addition, since entrepreneurs with better outside options tend to take larger risks as shown in Section 2 and 3, average productivity of the firms operated by those entrepreneurs will grow faster. In Figure 7, I plot growth in average productivity for firms operated by entrepreneurs with h in the 10th, 50th, and 90th percentile in the h distribution. The figure shows faster growth in average productivity for firms operated by a higher h.

Figure 7: Growth of Average Productivity by Firm Age with Different Outside Options



5.3 Counterfactual Exercises

5.3.1 Removing the Outside Option

To study the quantitative importance of outside options and the associated risk-taking behavior of young firms in the aggregate economy, I study a counterfactual situation in which entrepreneurs cannot return to paid employment. Though this is an extreme experiment, it provides a useful insight on how the existence of outside options affects the composition of startup firms and their life-cycle dynamics. It also provides an estimate of the upper bound of the output and productivity losses an economy can suffer from overall deterioration of the outside options of startup entrepreneurs.

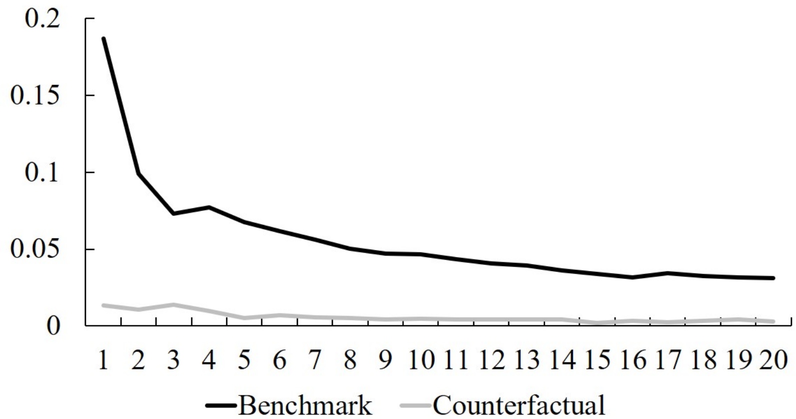
Table 11 shows a comparison of key statistics between the benchmark economy and the counterfactual economy. By construction, the firm exit rate falls to the exogenous death rate in the counterfactual case. Interestingly, the average productivity and size of entrants increase in the counterfactual economy. This is because if individuals know that they can never go back to paid employment, they will enter entrepreneurship only if their initial business productivity endowments are high enough. This positive selection effect, together with the steady-state force which equates the entry rate to the exit rate, induces the firm entry rate to fall. This result also indicates that an overall decline in outside options generates fewer but initially better startup firms.

Table 11: Overall Effect of Removing the Outside Option

	Benchmark	Counterfactual	% Changes
Firm entry rate	0.065	0.050	-23%
Firm exit rate (age 1-5)	0.104	0.050	-51.9%
Firm exit rate (age 6+)	0.055	0.050	-9.1%
Average entrant size (h)	5.775	6.576	13.9%
Average entrant z	4.403	5.314	20.7%
Aggregate output	2.797	2.547	-8.9%
Aggregate output per worker	1.438	1.375	-4.4%

However, aggregate output and labor productivity fall significantly in the counterfactual economy, by 8.9% and 4.4%, respectively. This is driven by differences in the growth rates of young firms along their life cycle. Because entrepreneurs in the counterfactual economy do not have an exit option to exercise in the case of business failure, they do not take risks early in their life cycle and thus show little or no growth as a group. On the other hand, although startups in the benchmark economy begin with lower productivity, they take larger risks and thus grow much faster as they age. Figure 8 illustrates this point by plotting the growth rates of the average productivity by firm age group. Therefore, this counterfactual experiment reveals that deterioration (or, for the same reason, improvement) of the outside options of startup entrepreneurs can have a sizable impact on aggregate output and productivity.

Figure 8: Growth of Average Productivity by Firm Age: Benchmark vs. Counterfactual



Notes: This figure shows the growth rate of average productivity by firm age in the benchmark economy and counterfactual economy where the outside options of the entrepreneurs are removed. Statistics are calculated from panel data with 100,000 startup firms obtained from the model simulation.

6 Conclusion

In this paper I show that the outside options of startup entrepreneurs, which I define as the level of labor income they expect to earn in the case of business failure, are a key predictor of the early growth trajectories of young firms. Better outside options serve as an effective channel of insurance against business failure, which enables entrepreneurs to take larger business risks. Larger risk-taking behavior translates into a more dispersed, up-or-out type of firm dynamics. I test these predictions using a large founder-firm matched administrative data set and find that the model implications are empirically supported.

I also show that large changes in the outside options of startup entrepreneurs can potentially have a large impact on aggregate output and productivity. An improvement in outside options induces smaller and less productive firms to enter, but incentivizes them to engage in riskier experimentation and exhibit faster average productivity growth along their life cycle. Therefore, the post-failure options of entrepreneurs are an important factor that governs not only young firm growth and survival, but also aggregate output and productivity growth.

The quantitative framework established in this paper can be extended to study several critical questions posed in macroeconomics. First, the model can be used to study whether the rise in labor income inequality in the U.S. during the last three decades have had an impact on the decline in high-growth entrepreneurship and business dynamism (Decker, Haltiwanger, Jarmin, and Miranda, 2016). While individuals with higher labor earnings are more likely to create high-growth young firms, it is possible that the rapid increase in their labor earnings may have made them less likely to enter entrepreneurship in the first place. Second, one can study the business cycle implications of this mechanism by introducing aggregate uncertainty and unemployment shocks in the model. When the aggregate economy is in a downturn and the unemployment rate is high, startup entrepreneurs would expect to experience difficulties finding a job if they cease their business operation. This would incentivize them to take less risks, further reducing young firm growth and thus resulting in lower levels of job creation in the economy. Therefore, risk-taking by startup entrepreneurs may work as a propagation mechanism of aggregate shocks.

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Appendix

A Simple Model Solution Derivation

Note that the entrepreneur in period 1 chooses labor n_1 to maximize period 1 profits and it does not affect period 2 expected utility. Explicitly writing out $V_1(z_1, h)$, the optimization problem of Δ can be re-written as

$$\max_{\Delta \geq 0} \ln(\Gamma z_1) + \beta \Big\{ e^{-\gamma\Delta} \cdot \max\{\ln(\Gamma z_1 e^{\Delta}), \ln(wh)\} + (1 - e^{-\gamma\Delta}) \cdot \max\{\ln(\Gamma z_1 e^{-\Delta}), \ln(wh)\} \Big\}$$

There are four possible objective functions depending on the realization of z_2 and the occupational choice. First, he can stay in business regardless of the z_2 realization, which delivers

$$V_1^{E,E}(z_1, h) = \max_{\Delta \geq 0} \ln(\Gamma z_1) + \beta \Big\{ e^{-\gamma\Delta} \cdot \ln(\Gamma z_1 e^{\Delta}) + (1 - e^{-\gamma\Delta}) \cdot \ln(\Gamma z_1 e^{-\Delta}) \Big\} \tag{17}$$

Second, he can stay in business in the high z_2 outcome and exit in the low z_2 outcome. In this case he gets

$$V_1^{E,W}(z_1, h) = \max_{\Delta \geq 0} \ln(\Gamma z_1) + \beta \Big\{ e^{-\gamma\Delta} \cdot \ln(\Gamma z_1 e^{\Delta}) + (1 - e^{-\gamma\Delta}) \cdot \ln(wh) \Big\} \tag{18}$$

Third, he can exit regardless of the z_2 realization, in which case he gets the value

$$V_1^{W,W}(z_1, h) = \max_{\Delta \geq 0} \ln(\Gamma z_1) + \beta \ln(wh) \tag{19}$$

Lastly, he can stay in business in the low z_2 and exit in the high z_2.

$$V_1^{W,E}(z_1, h) = \max_{\Delta \geq 0} \ln(\Gamma z_1) + \beta \Big\{ e^{-\gamma\Delta} \cdot \ln(wh) + (1 - e^{-\gamma\Delta}) \cdot \ln(\Gamma z_1 e^{\Delta}) \Big\} \tag{20}$$

One can solve the problem by first deriving the optimal Δ^* and the associated value functions conditioning on each case, and then finding the upper envelope of the conditional value functions over (z_1, h).

Case 1: $V_1^{E,E}(z_1, h)$

The Kuhn-Tucker theorem implies that the necessary conditions for the optimal Δ^* are

$$
\begin{aligned}
& -2\Delta^*\gamma e^{-\gamma\Delta^*} + 2e^{-\gamma\Delta^*} - 1 \leq 0 \\
& \big(-2\Delta^*\gamma e^{-\gamma\Delta^*} + 2e^{-\gamma\Delta^*} - 1\big) \cdot \Delta^* = 0 \\
& \Delta^* \geq 0
\end{aligned}
$$

Since $\Delta^* = 0$ violates the first condition, Δ^* is strictly positive. Thus Δ^* is the root of $-2\Delta^*\gamma e^{-\gamma\Delta^*} + 2e^{-\gamma\Delta^*} - 1 = 0$. Denote the solution as $\bar{\Delta}(\gamma)$. The implicit function theorem implies that $\bar{\Delta}$ is decreasing in γ.

Note that equation (17) can be re-written as

$$
V^{E,E}(z_1, h) = \ln(\Gamma z_1) + \beta\Big\{\ln(\Gamma z_1) + 2\bar{\Delta}e^{-\gamma\bar{\Delta}} - \bar{\Delta}\Big\}
$$

Define the second term inside of the bracket as $C(\gamma) = 2\bar{\Delta}e^{-\gamma\bar{\Delta}} - \bar{\Delta}$. $C(\gamma)$ is non-negative at the optimum. It is because if $C(\gamma)$ were negative, a higher objective function value can be achieved under $\Delta^* = 0$, which violates the first necessary condition.

Case 2: $V_1^{E,W}(z_1, h)$

The necessary conditions for the optimal Δ^* are

$$-\ln(\Gamma z_1) - \Delta^* + \frac{1}{\gamma} + \ln(wh) \leq 0$$
$$\big(-\ln(\Gamma z_1) - \Delta^* + \frac{1}{\gamma} + \ln(wh)\big) \cdot \Delta^* = 0$$
$$\Delta^* \geq 0$$

Define $\bar{h}(z_1) = \frac{e^{\frac{-1}{\gamma}}\Gamma z_1}{w}$. Note that $-\ln(\Gamma z_1) + \frac{1}{\gamma} + \ln(w\bar{h}(z_1)) = 0$. Then the optimal Δ^* can be characterized as

$$\Delta^* = \begin{cases} \ln(wh) - \ln(\Gamma z_1) + \frac{1}{\gamma} & h \geq \bar{h}(z_1) \\ 0 & h < \bar{h}(z_1) \end{cases}$$

Replacing Δ in equation (18) with Δ^*, the conditional value function is solved as

$$V^{E,W}(z_1, h) = \begin{cases} \ln(\Gamma z_1) + \beta\big\{\ln(wh) + \frac{1}{\gamma}(\frac{\Gamma z_1}{wh})^{\gamma} e^{-1}\big\} & h \geq \bar{h}(z_1) \\ \ln(\Gamma z_1) + \beta \ln(\Gamma z_1) & 0 \leq h < \bar{h}(z_1) \end{cases}$$

Case 3: $V_1^{W,W}(z_1, h)$

This case can be ignored as it is strictly dominated by $V^{E,W}$. Suppose $wh \geq \Gamma z_1$. Then $h > \bar{h}$ and thus $V^{E,W} > V^{W,W}$. On the other hand, if $wh < \Gamma z_1$, $V^{E,W} > V^{W,W}$ regardless of the value of h within the range.

Case 4: $V_1^{W,E}(z_1, h)$

This case can be ignored as it is weakly dominated by $V^{W,W}$ and $V^{E,E}$. Suppose $wh \geq \Gamma z_1$. Then $\ln(wh) - \ln(\Gamma z_1) \geq 0$. Subtracting $V^{W,E}$ from $V^{W,W}$, one can obtain $V^{W,E} - V^{W,W} = (1 - e^{-\gamma\Delta})\big(\ln(wh) - \ln(\Gamma z_1) + \Delta\big)$, which is weakly positive. On the other hand, suppose $wh < \Gamma z_1$. Then $\ln(\Gamma z_1) > \ln(wh)$. Subtracting $V^{W,E}$ from $V^{E,E}$, one can obtain $V^{E,E} -$

$V^{W,E} = e^{-\gamma\Delta}\big(\ln(\Gamma z_1) + \Delta - \ln(wh)\big)$, which is strictly positive.

Therefore it only requires comparing $V_1^{E,E}(z_1, h)$ and $V_1^{E,W}(z_1, h)$ to uncover the upper envelope and the optimal solution Δ^*. This can be done by fixing z_1 to an arbitrary value and varying the value of h. First, consider $h = 0$. At this point, $V^{E,W}(z_1, 0) = \ln(\Gamma z_1) + \beta\ln(\Gamma z_1)$ and $V^{E,E}(z_1, 0) = \ln(\Gamma z_1) + \beta(\ln(\Gamma z_1) + C(\gamma)$. Since $C(\gamma) > 0$, $V^{E,W}(z_1, 0) < V^{E,E}(z_1, 0)$.

Note that $\frac{\partial V^{E,E}(z_1,h)}{\partial h} = 0$, thus $V^{E,E}$ stays constant for all values of h. On the other hand, $\frac{\partial V^{E,W}(z_1,h)}{\partial h} = 0$ for $0 \leq h < \bar{h}(z_1)$, and $\frac{\partial V^{E,W}(z_1,h)}{\partial h} > 0$ for all $h > \bar{h}(z_1)$. Therefore as h moves from 0 to infinity, $V^{E,W}$ continuously increases starting from $\bar{h}$, and $V^{E,W}$ and $V^{E,E}$ crosses once and only once at a value $h^*(z_1) > \bar{h}(z_1)$. Therefore, the optimal Δ^* can be characterized by

$$\Delta^* = \begin{cases} \ln(wh) - \ln(\Gamma z_1) + \frac{1}{\gamma} & h \geq h^*(z_1) \\ \bar{\Delta}(\gamma) & 0 \leq h < h^*(z_1) \end{cases}$$

B Robustness Check: Sole-proprietor Sample

This section shows that the empirical evidence presented in section 3 are robust to restricting the sample to sole-proprietor firms. The purpose of this robustness analysis is to show an evidence that the results are not likely driven by potential errors in the founder approximation method. Table B1 reports the results from the linear probability regression where the dependent variable is the firm exit indicator. Table B2 reports the regression on firm-level growth dispersion, and Table B3 shows results for growth conditioning on survival. In contrast to the main regressions, I find insignificant coefficient for labor productivity growth while all other results are robust. Lastly, Table B4 shows results for the Hurst-Pugsley sector indicator interactions which shows less consistency for sole-proprietor firms.

Table B1: Firm Exit Regressions for Sole-Proprietors

	(1) Exit	(2) Exit	(3) Exit	(4) Exit
Log prior earnings	-0.015***	0.001	0.015***	0.016***
	(0.005)	(0.003)	(0.002)	(0.002)
Lagged log employment		-0.060***	-0.112***	-0.113***
		(0.002)	(0.003)	(0.004)
Lagged log labor prod.			-0.132***	-0.135***
			(0.007)	(0.008)
Lagged log wage				0.004
				(0.003)
Founder average age				-0.002***
				(0.000)
Founder male share				0.005
				(0.003)
Ind-Year FE	No	Yes	Yes	Yes
Firm age FE	No	Yes	Yes	Yes
State FE	No	No	No	Yes
Birth year FE	No	No	No	Yes
Obs.	450000	450000	450000	450000
R-sq	0.001	0.104	0.194	0.197

Notes: The table reports results for a linear probability regression where the dependent variable is firm exit indicator. The sample is restricted to sole-proprietors, whose business ownership information can be obtained from the Business Register (BR). Standard errors are clustered at the industry (NAICS4) level. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

Table B2: Growth Dispersion Regressions for Sole-Proprietors

	(1) ϵ^2 (Rev)	(2) ϵ^2 (Rev)	(3) ϵ^2 (Prod)	(4) ϵ^2 (Prod)
Log prior earnings	0.008***	0.007***	0.005***	0.004***
	(0.000)	(0.000)	(0.000)	(0.000)
Lagged log employment		0.002***		-0.003***
		(0.000)		(0.000)
Ind-Year FE	No	Yes	No	Yes
Firm age FE	No	Yes	No	Yes
Obs.				
R-sq	360000	360000	360000	360000
r2	0.001	0.023	0.000	0.021

Notes: The table reports results from estimating equation (7) for sole-proprietor sample. ϵ^2_{ijt} (Δ Rev) and ϵ^2_{ijt} (Δ Prod) are the squared deviations obtained from equation (6) where Y_{ijt} are revenue and labor productivity, respectively. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

Table B3: Growth Regressions for Sole-Proprietor Continuers

	(1) Δ Rev	(2) Δ Prod	(3) Δ Emp
Log prior earnings	0.009***	-0.003	0.012***
	(0.003)	(0.005)	(0.003)
Lagged log labor prod.	-0.078***	-0.198***	0.119***
	(0.006)	(0.007)	(0.007)
Lagged log employment	-0.049***	0.063***	-0.116***
	(0.004)	(0.006)	(0.008)
Lagged log wage	0.020***	-0.009***	0.032***
	(0.003)	(0.003)	(0.003)
Founder average age	-0.002***	-0.000***	-0.002***
	(0.000)	(0.000)	(0.000)
Founder male share	0.025***	0.0434***	-0.019***
	(0.003)	(0.005)	(0.004)
Ind-Year FE	Yes	Yes	Yes
Firm age FE	Yes	Yes	Yes
State FE	Yes	Yes	Yes
Birth year FE	Yes	Yes	Yes
Obs.	360000	360000	360000
R-sq	0.063	0.139	0.155

Notes: The table reports results for OLS regression of firm growth on prior earnings where the sample is restricted to the sole-proprietors. All growth measures are calculated as the DHS growth rate. Standard errors are clustered at the industry (NAICS4) level. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

Table B4: Regression Results with the Hurst-Pugsley Sector Interactions: Sole-Proprietor Sample

	(1) Exit	(2) Δ Rev	(3) Δ Prod	(4) Δ Emp	(5) ϵ^2 (Rev)
Log prior earnings	0.014***	0.009***	-0.003	0.011***	0.008***
	(0.001)	(0.002)	(0.004)	(0.003)	(0.001)
HP $\times$ log prior earnings	0.003	0.000	-0.001	0.001	-0.002***
	(0.002)	(0.004)	(0.008)	(0.005)	(0.001)
Lagged log labor prod.	-0.135***	-0.078***	-0.198***	0.119***	
	(0.008)	(0.006)	(0.007)	(0.007)	
Lagged log employment	-0.113***	-0.049***	0.063***	-0.117***	0.002***
	(0.004)	(0.004)	(0.006)	(0.008)	(0.000)
Lagged log wage	0.004	0.020***	-0.009***	0.032***	
	(0.003)	(0.003)	(0.003)	(0.003)	
Founder average age	-0.002***	-0.002***	-0.000**	-0.002***	
	(0.000)	(0.000)	(0.000)	(0.000)	
Founder male share	0.005**	0.025***	0.043***	-0.019***	
	(0.003)	(0.003)	(0.005)	(0.004)	
Ind-Year FE	Yes	Yes	Yes	Yes	Yes
Firm age FE	Yes	Yes	Yes	Yes	Yes
State FE	Yes	Yes	Yes	Yes	No
Birth year FE	Yes	Yes	Yes	Yes	No
Obs.	450000	360000	360000	360000	360000
R-sq	0.197	0.063	0.139	0.155	0.023

Notes: The table reports results for linear regressions re-estimated after including the interaction between HP indicator and log prior earnings. The sample only includes sole-proprietor firms. Standard errors are clustered at the industry level (NAICS4). ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.