Market Power in Small Business Lending

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Abstract

Bank lending is an important financing channel for small firms. Yet, banks in the U.S. have substantial market power and loan terms are unfavorable to borrowers in concentrated markets. What are the efficiency implications and policy remedies to bank concentration? We build a model of bank competition with endogenous interest rates, loan size, and take-up. We estimate the model using the universe of loans made through the Small Business Administration (SBA). Our identification strategy relies on 1) the excess bunching of loans around the discontinuity in SBA’s regulatory restrictions, and that 2) the excess bunching is more pronounced in concentrated markets. Despite federal subsidies through the SBA, preliminary results suggest market power causes 25% aggregate efficiency loss in small business lending. The optimal market-dependent interest rate subsidy can raise welfare by 15%.

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1 Introduction

Bank lending is an important financing channel for young and small firms and is therefore critically important for the aggregate economy (Kaplan and Zingales (1997)). Yet, reliance on geographic proximity between borrowers and lenders (Petersen and Rajan (1994)) can give banks substantial market power and potentially cause under-provision of credit (Dreschler, Savov, and Schnabl (2017)). Several federal programs in the United States exist to regulate pricing and encourage bank lending to small businesses.

To study this problem, we build and estimate a model of imperfect competition in bank lending with endogenous interest rates, loan size, and take-up. In the model, a finite number of banks compete for borrowers by offering loan contracts. Each contract specifies both the interest rate and the loan size. Banks are differentially preferred by borrowers with idiosyncratic taste shocks over banking services. Taste heterogeneity, together with the finiteness of competing banks, grants banks market power.

The model generates a mapping from bank concentration to lending outcomes and clarifies the implications of bank market power on both the intensive and extensive margins of bank lending. Despite market power, the intensive margin of loan size is always efficient conditioning on loan issuance, as banks choose the optimal loan size to maximize joint bank-firm surplus and only use interest rates to optimally extract surplus. However, market power distorts the extensive margin of lending, as high interest rates in concentrated local markets discourages firms from taking out loans. Our model is highly tractable: it yields analytic solutions and is amenable to theoretical normative policy analysis.

We estimate the model using the universe of small business loans made through a major federal loan subsidy program in the United States - the Small Business Administration (SBA) Express program. The SBA guarantees loans made by commercial lenders to in-need small businesses that are otherwise rejected from all other sources of external financing. It therefore relies on the existing banking infrastructure to pass the subsidy through to targeted firms.

Using a novel identification strategy that combines geographic variations in bank concentration and discontinuities in the regulatory specifications of the SBA program, we quantify the impact of market power on small business lending. We find quantitatively substantial market power and inefficiencies: on average, banks capture 20-30% of surplus from lending relationships. We estimate the efficacy of loan subsidy through the SBA program and perform a wide range of policy counterfactuals.

The coverage, granularity, and policy variation contained within this dataset makes it the ideal
laboratory to study market concentration. We observe the location of both borrowers and banks, which allows us to generate measures of market concentration both cross-sectionally and over time, as well as contract-level information on loan terms (interest rate, size) and repayment outcomes. Loans made through the program are subject to an interest rate ceiling, which places an exogenous constraint on banks’ pricing problems.

Our identification strategy builds on the literature that uses kinks and notches to identify key elasticities (Kleven (2016)), extending the methodology into a two dimensional space. Broadly speaking, this approach uses discontinuities in economic agents’ choice set and the consequent discontinuities in the equilibrium outcome distribution to infer structural parameters that govern economic behaviors. In our setting, loans made through the SBA are subject to a loan-size dependent interest rate cap — specifically loans smaller or equal to $50,000 are capped at a rate of Prime + 6.5%, while loans larger than $50,000 are limited to Prime + 4.5%.

Our model generates empirically testable predictions for 1) how lenders should respond to the specific policy variation in our dataset, and 2) how this response should differ across markets with varying degrees of competition. This regulation impacts equilibrium loan terms and generates excess mass in the distribution of loan size right before the interest ceiling becomes discontinuously more stringent. In more concentrated markets, banks have incentives to charge higher interest rates; consequently, the interest rate ceiling is more binding, and the excess mass generated by the regulatory discontinuity is more pronounced. Importantly, this excess mass is an empirically tractable object that maps to the theoretical moments of the model. We use the observed relationship between excess mass and market concentration to estimate the model parameters and understand how market power affects loan terms.

Methodologically, we advance the bunching approach to environments with multi-dimensional behavioral response. Existing papers that employ the bunching approach study how discontinuities in choice sets affect the equilibrium distribution of a single endogenous choice variable, relative to a counterfactual absent any policy discontinuity. In our setting, banks compete on two-dimensional loan contracts—loan size and interest rates—and respond in both dimensions to discontinuities in the interest ceiling. Furthermore, this means the loan distribution could become distorted both above and below the discontinuity. We provide general moment conditions for estimation that address these challenges of multi-dimensional behavioral response, and derive a methodology to recover the joint counterfactual distribution of loan and interest rate contracts.

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1For example Best and Kleven (2018) study how the UK mortgage market responds to transaction taxes, and study how labor supply responds to tax notches in Pakistan (Kleven and Waseem (2013)) and tax rate kinks in the US Saez (2010).
This paper relates to three literatures. First, we contribute to the broad literature on the rising market concentration in the United States (Barkai (2018), De Loecker (2018), Liu, Mian, and Sufi (2019)). Notably, the market power of financial institutions has been studied in the context of deposits (Dreschler et al. (2017), Dreschler, Savov, and Schnabl (2018)) and consumer lending (Nelson, Cuesta). We contribute to this literature by studying implications of concentration on bank lending, both theoretically and empirically.

Second, we contribute to the “bunching” literature (Kleven (2016), Best and Kleven (2018), Kleven and Waseem (2013), DeFusco and Paciorek (2017)) by advancing the methodology to multi-dimensional behavioral responses. We advance the bunching approach, which has traditionally focused on a single endogenous choice variable, to approaches with a multi-dimensional behavioral response. We apply this to a standard choice that banks and clients make—jointly determining interest rates and loan limits, but the approach can be used in a wide variety of contexts in economics and finance.

Third, we contribute to the literature on small business lending in the United States (Barrot (2016), Bachas, Kim, and Yannelis (2019), Brown and Earle (2017), Darmouni and Sutherland (2018), Evans and Jovanovic (1989), Granja, Leuz, and Rajan (2018), Howell (2017), Petersen and Rajan (1994, 1995), Kaplan and Zingales (1997)). This paper provides new evidence on how market power shapes small business lending, and how interest rate subsidies from a large federal program impact aggregate welfare.

We begin with an exposition of the model in section 2. Section 3 discusses the empirical setting, data, and relevant policy variation, while section 4 discusses the identification strategy. Section 5 describes the estimation procedure and empirical findings. Section 6 concludes.

2 Model

2.1 Setup

Consider a market with finite $K$ banks and a continuum of borrowers of finite measure. Let $k$ index for banks and $i$ index for borrowers.

Loan Demand and Borrower Payoff Each borrower is characterized by a tuple of characteristics, $\mathbf{x}_i \equiv (z_i, p_i, \delta_i)$. In the cross-section, these characteristics $\mathbf{x}_i$ follow a continuous joint distribution $F$ with bounded support. Borrower of characteristics $\mathbf{x}_i$ has a production function that
generates output from bank loans according to
\[
\begin{cases}
  z_i L^\alpha & \text{with probability } p_i, \\
  \delta_i L & \text{otherwise},
\end{cases}
\]
where $L$ is the size of loan, $z_i$ is his productivity, and $\delta_i < 1$ is the loan recovery rate in case of project failure.

A loan contract is a doublet $(r, L)$ of interest rate and loan size. We model default as involuntary: if a borrower accepts a contract, he gets $L$ from the bank to invest into his project, and he repays $(1 + r) L$ when his project succeeds and $\delta_i L$ when it fails. Borrower $i$’s utility of taking out a loan from bank $k$ is
\[
u_{ik} = \xi_{ik} \times p_i (z_i L^\alpha - (1 + r) L),
\]
where $\xi_{ik} \geq 0$ is a random taste shock that is drawn from distribution $G_K$ and is i.i.d. across borrowers and banks. We refer to $\nu_{ik} \equiv p_i (z_i L^\alpha - (1 + r) L)$ as the contractual utility and $\nu_{ik}$ as the realized utility. The taste shocks $\xi_{ik}$ represent idiosyncratic heterogeneity, such as borrowers’ differential preferences for the services provided by differentiated banks. Every borrower can choose not to take out any loans. We normalize the utility of this outside option to be another random realization of the idiosyncratic shock $\xi_{i0}$ drawn from $G_K$. We elaborate on properties of these shocks below.

Without loss of generality, every bank offers one contract to each borrower. The borrower’s problem is to choose the contract $k$ that yields him the highest realized utility $\nu_{ik}$. Let $\nu_{i0} \equiv 1$, and we say the borrower chooses contract $k = 0$ if he decides not to borrow. The probability that borrower $i$ chooses bank $k$, which we denote as $q_{ik}$, is
\[
q_{ik} \equiv Pr(i \text{ chooses } k) = Pr(\xi_{ik} \nu_{ik} \geq \xi_{ik'} \nu_{ik'} \text{ for all } k').
\]
$q_{ik}$ is increasing in $\nu_{ik}$, the attractiveness of the contract offered by bank $k$, and decreasing in $\nu_{ik'}$ for all $k' \neq k$. More generally, we can write the choice probability as a function of contractual utilities: $q_{ik}(\nu_{ik}; \nu_{ik'})$. We normalize $\xi_{ik}$ so that the expected value of realized utility is equal to the average contractual utility ($E[\max_k \{\xi_{ik} \nu_{ik}\}] = \sum_{k=0}^{K} q_{ik} \nu_{ik}$).

5
Loan Supply and Lender Payoff  For a given borrower $i$, each lender chooses $(r, L)$ to maximize expected profit, taking other lenders’ loan terms as given. Bank $k$’s expected surplus is

$$LS_{ik} \equiv \max_r [p_i (1 + r_{ik} - \delta_i) L_{ik} - (\bar{c} - \delta_i) L_{ik}] q \left( v_{ik}; \{v_{ik'}\}_{k' \neq k} \right) \quad \text{s.t. } v_{ik} = p_i (z_i L_{ik}^\alpha - (1 + r_{ik}) L_{ik}).$$

(3)

The term $\bar{c}$ represents each lender’s unit cost of lending, which include both the opportunity cost of funds as well as any monitoring and servicing costs associated with lending. Given the repayment probability $p_i$ and the recovery rate $\delta_i$, the entire expression in the bracket represents bank $k$’s expected profits—revenue net of costs—if the loan is accepted. For expositional purposes, we model banks as symmetric and have the same unit cost of lending $\bar{c}$. Our theory easily extends to cases with asymmetric costs.

Definition 1. A Laissez-faire equilibrium of a market with $K$ lenders is defined as the collection of loan terms and choice probabilities $\{(r_{ik}, L_{ik}), q_{ik}\}_{i,k}$ such that $q_{ik}$ satisfies (2), and loan terms solve bank’s problem (3).

Note that the idiosyncratic shocks $\xi_{ik}$ serve as a modeling tool to generate taste differences. When these shocks are drawn from a degenerate distribution (e.g. $\xi_{ik} = 1$ for all $i,k$), all banks are perfect substitutes. Conversely, banks can be highly differentiated if the underlying distribution $G_K$ exhibits a high variance.

2.2 The Laissez-faire Equilibrium

The solution to bank $k$’s profit maximization problem in (3) coincides with the solution of the following problem

$$\max_{r_{ik}, L_{ik}} [(1 + r_{ik}) - c_i] L_{ik} \times q_{ik} \left( v_{ik}; \{v_{ik'}\}_{k' \neq k} \right) \quad \text{s.t. } v_{ik} = p_i (z_i L_{ik}^\alpha - (1 + r_{ik}) L_{ik})$$

(4)

where $c_i \equiv \bar{c} - \delta_i (1 - p_i) / p_i$. Intuitively, $c_i$ is the effective marginal cost of lending to borrower $i$, after taking into account 1) bank’s cost of funds $\bar{c}$, 2) repayment probability $p_i$, and 3) recovery rate $\delta_i$ in case of default.

Let $\varepsilon_{ik} \equiv \partial \ln q_{ik} / \partial \ln v_{ik}$ be the elasticity of $q_{ik}$ with respect to $v_{ik}$, holding contracts offered by all other banks constant. We refer to $\varepsilon_{ik}$ as the demand elasticity with respect to the contractual utility provided by bank $i$. It is non-negative and stands in contrast to the standard demand elasticity in price theory, which is a derivative taken with respect to prices and is always non-positive.
The first-order conditions of (4) over the loan size and interest rate are, respectively,

\[
\{L_{ik}\} \quad \frac{\alpha z_i L_{ik}^\alpha - (1 + r_{ik}) L_{ik}}{z_i L_{ik}^\alpha - (1 + r_{ik}) L_{ik}} = -\frac{1}{\varepsilon_{ik}}, \tag{5}
\]

\[
\{r_{ik}\} \quad \frac{(1 + r_{ik} - c_i) L_{ik}}{z_i L_{ik}^\alpha - (1 + r_{ik}) L_{ik}} = \frac{1}{\varepsilon_{ik}}. \tag{6}
\]

**Proposition 1.** *In the Laissez-faire equilibrium, loan terms satisfy*

\[
L_{ik} = \left( \frac{\alpha z_i}{c_i} \right)^{\frac{1}{1-\alpha}}, \tag{7}
\]

\[
\frac{1 + r_{ik} - c_i}{c_i} = \frac{1 - \alpha}{\alpha} \frac{1}{\varepsilon_{ik} + 1}. \tag{8}
\]

The proposition characterizes equilibrium loan size and interest rate. Equation (7) shows that equilibrium loan size is efficient, as \(L_{ik}\) solves the maximization problem \(\max_L p_i z_i L^\alpha + (1 - p_i) \delta_i L - c_i L\). This result may come as a surprise: banks do have market power, as they are profit-maximizing entities that choose contractual terms. Why is there no distortion over loan provision along the intensive margin? The intuition is that each bank correctly calculates the benefit and cost of lending. Hence, conditioning on \((i,k)\) forming a relationship, bank \(k\) chooses the most efficient loan size and extract rent only by charging an interest rate that is higher than the marginal lending cost.

The equilibrium interest rate is characterized by equation (8). The left-hand-side captures each bank’s profit margin (profit relative to cost). Equilibrium margin can be written as a function of \(\alpha\), the concavity of borrower’s production technology, together the demand elasticity \(\varepsilon_{ik}\). To understand the expression, note that \(z_i L_{ik}^\alpha - c_i L\), the total surplus generated by a loan of size \(L\), is equal to \(\frac{1 - \alpha}{\alpha} c_i L\) when the loan size is optimally chosen. Equation (7) therefore implies bank’s profit, relative to total surplus generated by loan \((r_{ik}, L_{ik})\), is equal to \(\frac{1}{\varepsilon_{ik} + 1}\). Bank’s share of surplus is lower when in more elastic markets.

How should interest rates differ between concentrated and competitive markets? Intuitively, when there are more competing banks, demand for loans from a specific bank should become more elastic, as a marginal increase in interest rate—and the consequent reduction in contractual utility—should lead to a greater outflow of potential borrowers. That demand should be more elastic in competitive markets also corresponds well with the formula (8) and the notion that interest rate and bank’s surplus should be lower in these markets.
**Distributional Assumption over $\xi_{ik}$** We now impose distributional assumption over the idiosyncratic taste shocks $\xi_{ik}$. The value of specializing the model is to obtain analytic solutions over equilibrium contracts. Our assumption generates the property that, ceteris paribus, demand is more elastic in more competitive markets.

**Assumption 1.** $\xi_{ik}$’s are drawn from a Frechet distribution, with CDF

$$G_K(\xi; \sigma) = e^{-(\gamma \xi)^{-\sigma} / (K+1)},$$

where $\gamma \equiv \Gamma(1 - 1/\sigma)$ is a normalizing constant and $\Gamma$ is the Gamma function (Johnson and Kotz (1970)).

Under Assumption 1, the choice probability for any given bank becomes

$$q_{ik} \left( \{v_{ik'}\}_{k'=0}^{K} \right) = \frac{\sum_{k'=0}^{K} v_{ik}^{\sigma}}{\sum_{k'=0}^{K} v_{ik'}^{\sigma}}.$$

(9)

Correspondingly, the probability that firm $i$ is unfunded—i.e. the firm chooses the outside option—is

$$q_{i0} = \frac{1}{\sum_{k'=0}^{K} v_{ik'}^{\sigma}}.$$

The expected utility of borrower $i$ is

$$EU_i = \left( \frac{1}{K+1} \sum_{k=0}^{K} v_{ik}^{\sigma} \right)^{\frac{1}{\sigma}}.$$

$\sigma > 0$ is an important parameter. It captures the substitutability of loans across banks and it relates inversely to the variance of the idiosyncratic taste shocks. Banks are more substitutable when $\sigma$ is high. As we show below, in the limit as $\sigma \to \infty$, banks become perfect substitutes. Conversely, as $\sigma \to 0$, all banks effectively become monopolists, as the choice probability converges to $\frac{1}{K+1}$ regardless of the contractual utilities $\{v_{ik'}\}$.

The demand elasticity under Assumption 1 is

$$\varepsilon_{ik} = \sigma (1 - q_{ik}).$$

Each bank’s profit margin is

$$\mu_{ik} \equiv \frac{1 + r_{ik} - c_i}{c_i} = \frac{1 - \alpha}{\alpha} \frac{1}{\sigma (1 - q_{ik}) + 1}.$$
Because all banks are symmetric, they offer the same contract, and the choice probability is \( q_{iK} = \frac{1-q_{i0}}{K} \).

**Proposition 2.** Under Assumption 1, the model has the following predictions about Laissez-faire.

1. **(Contractual Terms)** The interest rate offered to firm \( i \) is lower when
   
   (a) the market is more competitive (higher \( K \)), and
   
   (b) when banks are more substitutable (higher \( \sigma \)).

   Conditioning on getting funded, loan size is efficient and is independent of \( K \) and \( \sigma \).

2. **(Extensive Margin)** A smaller share of firms remain unfunded in more competitive markets (\( q_{i0} \) is decreasing in \( K \)).

3. **(Differential Impact of Market Power for Productive and Unproductive Firms)** Consider firm \( i \) and \( j \) with \( z_i/c_i > z_j/c_j \), then

   \[
   q_{j0}(K;\sigma) - q_{j0}(K+1;\sigma) > q_{i0}(K;\sigma) - q_{i0}(K+1;\sigma),
   \]

   \[
   -\frac{dq_{j0}(K;\sigma)}{d\sigma} > -\frac{dq_{i0}(K;\sigma)}{d\sigma}.
   \]

   In words, if firm \( i \) is effectively more productive than firm \( j \) (measured by using productivity \( z \) relative to the effective lending cost \( c \)), then firm \( j \) benefits more along the extensive margin (i.e. getting funded with higher probability, \( q_{i0} \) declines) than firm \( i \) as the lending market becomes more competitive (\( K \) or \( \sigma \) increases).

To summarize, market power translates into high interest rates, which distorts the extensive margin of lending, but does not affect the intensive margin of loan size. Moreover, the extensive margin is more distorted by market power for less productive contracts, as they are more likely to reject all contracts.

The theory also relates the average profit margin in a market to its Herfindahl index (HHI). Let

\[
s_k \equiv \frac{\int q_{ik}L_{ik}dF_i}{\sum_{k'} q_{ik'}L_{ik'}dF_i}
\]

be the market share of bank \( k \) weighted by loan size, where the integration is taken over the distribution of borrower characteristics \( F \). Likewise, define the average profit margin in a market as

\[
\mu = \frac{\int \mu q_{ik}L_{ik}dF_i}{\sum_{k'} q_{ik'}L_{ik'}dF_i}.
\]

**Proposition 3.** In the Laissez-faire equilibrium, the average profit margin in a market can be written as

\[
\mu = \frac{1 - \alpha}{\alpha (1 + \sigma)} + \frac{(1 - \alpha)}{\alpha (1 + \sigma) (1 + \sigma)} \times \text{HHI} + o \left( \max_k (s_k)^2 \right)
\]
The term $o(\cdot)$ vanishes as $K \to \infty$.

The proposition shows that the profit margin is approximately linearly increasing in market concentration, as measured by the $HHI$.

### 2.3 Policy Intervention

The goal of our paper is to quantify the efficiency cost of market power and to understand how policies can remedy such inefficiencies. Our identification strategy relies on discontinuities in policy constraints over loan terms. Specifically, our empirical setting involves size-dependent interest rate caps. In this section, we analyze, theoretically, how do equilibrium contracts respond to policy constraints over loan terms.

Because contracts are two-dimensional, imposing any binding constraints over one of the choice variables $r$ and $L$ will intuitively cause banks to respond over the other choice variable. Specifically, the first-order conditions (5) and (6) imply that the two choice variables $r$ and $L$ serve as strategic substitutes in each bank’s profit maximization problem. That is, holding demand elasticity $\varepsilon_{ik}$ constant, equation (5) shows that the optimal choice of $L$ is a decreasing function of $r$, and (6) shows that the optimal choice of $r$ is a decreasing function of $L$. Hence, imposing a binding interest rate cap leads banks to over-lend, as loan size becomes larger than what’s efficient; likewise, imposing a binding loan size cap leads banks to charge higher interest rates than what would have prevailed under Laissez-faire.

The property, that $r$ and $L$ are strategic substitutes, holds in our model with and without Assumption 1. Under the assumption, we are further able to provide analytic solutions to the contractual response under various policy constraints. For notational ease, because all banks are symmetric and offer identical contracts, we drop the subscript $k$ whenever it’s unambiguous.

**Proposition 4.** Consider bank’s profit maximization problem (4) under additional constraints. Let $(r^*_i, L^*_i)$ represent the Laissez-faire contract.

1. The equilibrium contract under the constraint $r_i \leq \tilde{r}$ is

   $$(r_i, L_i) = \left( \min \{\tilde{r}, r^*_i\}, L^*_i \times \max \left\{1, \left( \frac{1 + r^*_i}{1 + \tilde{r}} \right)^{\frac{1}{1-\alpha}} \right\} \right).$$
2. The equilibrium contract under the constraint $L_i \leq \bar{L}$ satisfies

$$\left(1 + r_i - c_i, \frac{c_i}{L_i}\right) = \left(z_i \min \{\bar{L}, L_i^*\} \frac{\alpha - 1}{c_i - 1} \left(1 + \frac{1 - q_i \alpha}{K}\right), \min \{\bar{L}, L_i^*\}\right).$$

Size-Dependent Interest Rate Caps Now consider size-dependent interest rate caps: there is an interest ceiling $\bar{r}_H$ for loans size below $\bar{L}$ and ceiling $\bar{r}_L < \bar{r}_H$ for loan size above $\bar{L}$. We continue to use $(r_i^*, L_i^*)$ to represent the Laissez-faire contract and use $(r_i, L_i)$ to represent the equilibrium contract under the policy.

Because the two choice variables $r$ and $L$ are strategic substitutes, we can intuitively categorize each bank’s response to the size-dependent interest rate cap into three scenarios, depending on borrower $i$’s characteristics and the policy environment $(\bar{r}_L, \bar{r}_H, \bar{L})$:

A. $r_i^* < \bar{r}_L$, or $(r_i^* < \bar{r}_H$ and $L_i^* < \bar{L}$): for these borrowers, the interest rate ceilings do not bind.

B. $r_i^* > \bar{r}_H$ and $L_i^* < \bar{L}$: for these borrowers, equilibrium loan terms have two possibilities:

(a) $L_i \leq \bar{L}$ and $r_i = \bar{r}_H$;

(b) $L_i > \bar{L}$ and $r_i = \bar{r}_L$.

C. $r_i^* > \bar{r}_L$ and $L_i^* \geq \bar{L}$: for these borrowers, equilibrium loan terms have two possibilities:

(a) $L_i > \bar{L}$ and $r_i = \bar{r}_H$;

(b) $L_i = \bar{L}$ and $r_i \in (\bar{r}_L, \bar{r}_H]$. 

\[ \text{Interest Rate} \]
\[ \text{Loan Size} \]

- Contract under Laissez-faire
- Possible contracts under interest rate ceiling

\[ r_H \]
\[ r_L \]
\[ \bar{L} \]
The next proposition formally characterizes the equilibrium contract.

**Proposition 5.** Suppose the Laissez-faire contract \((r_i^*, L_i^*)\) is infeasible under the policy environment with rate cap \(\bar{r}_H\) for \(L < \bar{L}\) and \(r_L^*\) for \(L > \bar{L}\). Let \((r_i^L, L_i^L) \equiv \left( r_i^L, L_i^L \left( \frac{1+r_i^L}{1+r^L} \right)^{\frac{1}{1-\alpha}} \right)\), and let

\[
(r_i^H, L_i^H) \equiv \begin{cases} 
(r_i^H, \min \left\{ L, L_i^* \left( \frac{1+r_i^L}{1+r^L} \right)^{\frac{1}{1-\alpha}} \right\} ) & \text{if } L_i^* < \bar{L} \\
\min \left\{ r_i^H, z_i \frac{L^{\alpha-1} + c_i (1-q_i^H) / K}{1 + (1-q_i^H) / K} - 1 \right\}, \bar{L} & \text{if } L_i^* \geq \bar{L}.
\end{cases}
\]

The equilibrium contract under rate ceilings is either \((r_i^L, L_i^L)\) or \((r_i^H, L_i^H)\). The equilibrium contract is \((r_i^H, L_i^H)\) if and only if

\[
V_i \equiv \frac{(1+r_i^H-c_i) L_i^H (1-q_i^H)}{(1+r_i^L-c_i) L_i^L (1-q_i^L)} > 1,
\]

where \(q_i^L\) and \(q_i^H\) are, respectively, the probability of outside option being chosen when contract \((r_i^L, L_i^L)\) and \((r_i^H, L_i^H)\) are offered.

For borrowers whose Laissez-faire contract is infeasible under the interest rate ceilings, bank either offer smaller loans with higher interest rates, \((r_i^H, L_i^H)\), or larger loans with lower interest rates, \((r_i^L, L_i^L)\). The object \(V_i\) represents the relative bank payoff between offering high interest rate (and small) loan and offering low interest rate (and large) loan. Marginal borrowers are those with \(V_i = 1\).

**Proposition 6.** Ceteris paribus, \(V_i\) is increasing in \(c_i\) and is decreasing in \(z_i\) and \(K\).

The proposition reveals that, under a size-dependent interest rate cap, banks are more likely to offer inefficiently small loans (higher \(V_i\)) to borrower \(i\) when
1. the market is more monopolistic (lower $K$),

2. the borrower is less productive (lower $z_i$), less likely to succeed (lower $p_i$), or has a lower recovery rate (lower $\delta_i$).

3 Institutional Background and Data

3.1 SBA Express Loans

The Small Business Administration (SBA) is an independent federal government agency tasked with the mission of providing assistance to small business. The agency provides commercial lenders with an indirect guarantee on loans made to targeted groups of borrowers, namely small businesses who document that they have been turned down for alternative forms of credit. Lenders pay a fixed guarantee fee (typically 1 to 3% of loan principal) to the SBA in return for a guarantee that the SBA will reimburse a certain percentage of loan principal in the case of default.

SBA Loans are intended as loans of last resort, to small businesses that cannot apply for credit elsewhere. To this end, firms that take out SBA loans must pass credit elsewhere checks, which aim to demonstrate that borrowers cannot take out loans at reasonable terms elsewhere. SBA lenders include both large banks, such as Wells Fargo, and specialized lenders such as Live Oak which almost exclusively offer SBA guaranteed loans.

We study the SBA Express Loan program, which provides expedited loans to small businesses at a higher cost. The program provides small businesses with long-term working capital financing up to $350,000. Loans made through the SBA guarantee program are subject to specific rules and regulations, the most important of which for this paper is the interest rate cap (described in detail below). More information on the SBA Express program is provided in Appendix B.

3.2 Data

Our dataset contains the universe of small business loans made through the Small Business Administration’s Express lending program from 2008-2018. We observe contract-level information on loan terms (interest rate, size) and repayment outcomes, borrower identity and characteristics and bank identity. The dataset also includes the location of both borrowers and banks, which allows us to generate measures of market concentration both cross-sectionally and over time. Borrowers in the SBA market must also document that they have been turned down for other forms of

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2 The SBA monitors this requirement with credit elsewhere checks, and failure to comply with these tests can lead to sanctions and exclusion from SBA programs.

3 While the review process is expedited, but funding time can still take up to 90 days.
credit; this creates a clearly defined market of banks (regional SBA lenders) for that particular borrower, and rules out the possibility that borrowers are “topping up” their SBA loans with additional sources of credit.

Table 1 presents summary statistics for our Express loan sample, which includes 240,188 loans made under the SBA Express program between 2008 and 2018. On average, these loans are $71,925 in size, and have a maturity 6.5 years. Interest rates for SBA Express loans can be fixed or variable; they are tied to base rates, with the maximum allowable interest rate ranging from 4.5 to 6.5 percent above the base rate, depending on loan size. The average interest rate in our sample is 6.9%, well above typical rates for corporate loans.

### 3.3 Motivating Facts

Despite the fact that the SBA lending market is heavily regulated, we still observe strong suggestive evidence in the data of imperfect competition. We calculate an inverse Herfendahl-Hirschman Index (HHI) based on the dollar volume lending share \(s\), of each bank within a given county-year:

\[
\text{InverseHHI}_{c,t} = \frac{1}{\sum_{i=1}^{N} s_{i,c,t}^2}.
\]

A value of 1 means that a single bank holds the entire market share, whereas larger values signal less market concentration. In a market where banks have equal market shares, the inverse HHI is simply the number of banks in the market. Figure 1, which plots the distribution of \(\text{InverseHHI}_{c,t}\), suggests that over 45% of county-year markets are monopolistic and only a small minority of markets have a substantial level of lender competition.

We also observe an impact of market concentration on loan pricing. Figure 2 documents a strong negative relationship between the average initial interest rate charged on observationally identical

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4These base rates are the prime rate, the LIBOR, and the PEG, which can fluctuate based on market conditions. For variable rates, the base rate for the computation of interest rate is the lender’s choice, provided that the maximum interest rate the borrower is charged still does not exceed prime rate plus 4.5 percent to 6.5 percent. It should also be similar to the rates the lender charges for other, similarly-sized, non-SBA guaranteed loans.
loans\textsuperscript{5} within a zipcode, and the number of lenders competing within that zipcode. A similar relationship exists for other measures of market concentration (e.g. county or zipcode HHI, banks within an X-mile radius). The right hand panel plot average ex-post losses on these loans across the same measure of market concentration, and suggests this negative relationship is not driven by differences in borrower risk across markets.

\textbf{3.4 Policy Variation}

While the trends in HHI and interest rates shown above motivate an analysis of market power, they remain suggestive; an exogenous shift or shock to lenders’ maximization problem is required to separate and identify the relevant demand and supply parameters. Loans made through the SBA Express program are subject to specific SBA rules and regulations that provide this identifying policy variation. Loans are subject to a loan-size dependent interest rate cap — specifically loans smaller or equal to $50,000 are capped at Prime + 6.5\%, while loans larger than 50,000 are limited to Prime + 4.5\%. This “notch” in the interest rate cap imposes a size-dependent constraint on banks’ pricing problem, and generates specific lending and pricing responses under our model of imperfect competition. SBA regulations do not allow lenders to originate multiple loans to the same borrower at the same time. Thus lenders cannot "piggyback" loans to take advantage of the notch.

\textsuperscript{5}We control for bank brand (i.e. West America, Chase, etc), borrower business NAICS code, loan maturity, and time fixed effects.
3.5 Reduced Form Evidence

Applied to this specific data and setting, our model specifies for 1) how lenders should respond to the specific policy variation in our dataset, and 2) how this response should differ across markets with differing degrees of competition. Importantly the theoretical response of lenders within and across markets has testable implications for the observed distribution of \((L, r)\) contracts.\(^6\)

Under constrained profit maximization, banks view loan size \((L)\) and interest rate \((r)\) as substitutes. Imposing a constant interest rate cap will suppress the interest rate for loans that bind, and consequently banks will substitute by increasing \(L\). An interest rate cap with an \(L\)-dependent “notch”, the variation found in our setting, generates a more nuanced response - impacted contracts from both the left and right right of the notch will move towards the threshold. Relative to the counterfactual loan and interest rate distribution absent policy variation, this will generate excess mass in the loan distribution at the threshold. As market concentration increases, loan markups will increase and a larger portion of loans will be impacted by the cap. This means that the size of the

\(^6\)In Appendix A figure 8 we use a calibrated version of our model to simulate both the response of the loan distribution under an interest rate cap notch, and that response across markets of varying size. The results of this simulation are very similar to the distributional responses that we observe in the data.
Figure 3: Interest Rate Ceiling and Average Interest Rate (Minus the Base Rate) as a function of loan size

excess mass will increase in market concentration.

Both of these predictions are borne out in the data. Figure 4 plots the distribution of $L$ separately for loans that have interest rates below the lowest interest rate cap ($r < \bar{r}_L$) and loans with rates at or above the lowest cap ($r \geq \bar{r}_L$). The intuition behind this grouping is that loans with ($r < \bar{r}_L$) are unaffected by the interest rate regulation, and therefore will have an undistorted loan size distribution. This provides an approximate “counterfactual” distribution\(^7\) that can be used to gauge the size of the excess mass at the threshold in the affected loan distribution for whom ($r \geq \bar{r}_L$).

Both groups exhibit an approximately log-normal loan distribution, and pronounced bunching at multiples of “round” numbers, e.g. multiples of $10,000$, $25,000$, and $50,000$. However, the loans impacted by the interest rate policy have additional mass ($\approx 5pp$) at the $50,000$ threshold where the cap discontinuously drops.

In line with the model predictions, this excess mass varies systematically with the degree of competition. Figure 5 shows that the average excess mass calculated at the market-zipcode level decreases with the number of banks within that zipcode, disappearing almost entirely in markets with greater than 20 lenders.

\(^7\)In section 5.1 we formally define and estimate a counterfactual distribution.
Figure 4: Loan size distribution for loans unaffected (left) and affected (right) by the interest rate cap.

Figure 5: Relationship Between Excess Mass and Market Concentration
4 Identification

We now discuss how to recover the structural parameters ($\alpha, \sigma$) from the interest rate cap. Our key identification strategy relies on the fact that banks respond to the policy intervention differentially in concentrated versus competitive markets. In more concentrated markets, banks have incentives to charge higher interest rates; consequently, the interest rate ceiling is more binding, and the excess mass generated by the regulatory discontinuity is more pronounced. Importantly, this excess mass is an empirically tractable object that maps to the theoretical moments of the model. We use the observed relationship between excess mass and market concentration to estimate the model parameters and understand how market power affects loan terms.

Intuitively, our strategy can be understood using the three figures below. The first figure shows the counterfactual distribution of loan size under Laissez faire. The second figure shows the equilibrium distribution of loan size under interest rate cap in a relatively competitive market, with 10 banks. There is excess bunching of loans right before the discontinuity in the interest rate cap. The third figure shows that the excess bunching is more pronounced in a concentrated market with 3 banks.

Our setting differs from a standard “bunching” approach in the literature (Kleven and Waseem 2013) because contracts are two-dimensional. Broadly speaking, the standard approach uses discontinuities in economic agents’ choice set over a single choice variable $x$ and the consequent discontinuities in the distribution of single-dimensional outcome to infer structural parameters that govern economic behaviors. The approach requires two steps: 1) recover a counterfactual distribution of equilibrium outcome $H^0(x)$ absent the policy discontinuity; 2) use the difference between $H^0(x)$ and the observed, equilibrium distribution under policy, $H^P(r,L)$, to infer a structural parameter.

To execute the identification strategy—mapping geographic variation in market concentration $K$ and the excess bunching of loan contracts to structural parameters ($\alpha, \sigma$)—we need to overcome two challenges. The first challenge is to recover the counterfactual distribution $H^0(x)$ of two-dimensional Laissez-faire contracts. Because banks respond to the interest rate cap by changing
both interest rates and loan size, contract distribution could become distorted both above and below the discontinuity in our setting.

The second challenge is the selection of moment conditions. How \( H^0(x) \) differs from the equilibrium distribution of contracts \( H^P(r,L) \) is governed by both structural parameters \((\alpha, \sigma)\) and market concentration \(K\). Because the joint distributions are continuous over \((r,L)\), there are, in principle, uncountably many moments—based on conditional distribution of loan size or interest rates—that can be used for estimation, i.e., the model is over-identified. How do we select moments in an economically transparent yet meaningful way?

We overcome the first challenge by recovering the counterfactual distribution \( H^0(r,L) \) from the conditional distribution of contractual terms from the subsample of loans with interest rates below the lower interest rate cap. That is, we recover \( H^0(r,L) \) from the conditional distribution of loans \( H^P(r,L|r < \bar{r}_L) \). This can be done under the following assumptions.

**Assumption 2.** 1) \( H^P(r,L|r < \bar{r}_L) = H^0(r,L|r < \bar{r}_L) \). 2) \( H^0(\cdot) \) is analytic over its domain.

The first part states that the size-dependent interest rate cap does not affect contracts for which the Laissez-faire interest rate is lower than \( \bar{r}_L \). The second part enables us to recover the full counterfactual distribution \( H^0(\cdot) \) from the conditional distribution \( H^0(\cdot|r < \bar{r}_L) \). We implement the strategy by estimating \( H^0(\cdot|r < \bar{r}_L) \) using a polynomial and then obtain \( H^0(\cdot) \) by extrapolation using the estimates.

For moments, we calculate the excess mass of loan contracts in the set \( B \equiv \{(r,L) | r \in (\bar{r}_L, \bar{r}_H), L = \bar{L}\} \) under distribution \( H^P \). Note that these are contracts that have loan size exactly at the cutoff \( \bar{L} \) and have interest rate strictly between the lower and upper caps.

Under Laissez-faire, the mass of contracts in set \( B \) should be zero. In equilibrium, the mass is strictly positive due to bunching. We form moment conditions based on this set of contracts because, in general, \( H^P \) is distorted from \( H^0 \) in both dimensions: loan size and interest rates can be distorted both upwards and downwards. In this region, however, all excess mass comes from Laissez-faire contracts with \( L^*_i > \bar{L} \), as shown in the illustrative figure below.
Formally, let
\[
S_1^K \equiv \left\{ (r_i, L_i) \left| \left( \frac{L/L_i}{1+(1-1/K)\sigma} \right)^\alpha - \alpha \left( \frac{1+r_i}{1+r_{L_i}} \right)^{1/\sigma}, L_i > L \right. \right\}
\]
\[
S_2^K \equiv \left\{ (r_i, L_i) \left| (1+r_i) \frac{(L/L_i)^{\alpha-1} + (1-(1-q_i)/K)\sigma\alpha}{1+(1-(1-q_i)/K)\sigma\alpha} < 1+r^H, L_i > \bar{L} \right. \right\}
\]

The intersection \( S_1^K \cap S_2^K \) corresponds to the blue region in the figure above, and Laissez-faire contracts \( (r^*, L^*) \in S_1^K \cap S_2^K \) will bunch into region \( B \) under the size-dependent interest rate cap. Intuitively, \( S_1^K \) picks out the Laissez-faire contracts that scale back to \( \bar{L} \) (instead of over-lending), and \( S_2^K \) picks out Laissez-faire contracts charge less than \( r^H \) under the policy intervention.

For each market structure \( K \), we generate the following moment condition
\[
\int \int_{(r,L) \in B} dH^P (r,L) = \int \int_{(r,L) \in S_1^K \cap S_2^K} dH^0 (r,L).
\]

We estimate \((\sigma, \alpha)\) by exploiting the moment condition across \( K \). Intuitively, the parameter \( \alpha \) captures the excess mass in set \( B \) when there is a single bank in the market, and the parameter \( \sigma \) captures how the excess mass in \( B \) varies with the number of banks, \( K \). Under this approach, both parameters are non-parametrically identified.

5 Estimation and Results

The excess mass we observe in the loan size distribution, as well as the “missing” mass to the left and right of the notch, are empirically tractable objects that map to theoretical moments of the model and allow for estimation of the model parameters. Here we describe the procedure for both retrieving the counterfactual distribution that allows us to measure the excess mass, and estimating the model parameters using a set of indifference conditions that equate the excess and missing mass in our two-dimensional setting.

5.1 Counterfactual loan and interest rate distribution

While the raw distribution of loan terms is suggestive of bunching at the interest cap threshold, a more precise measure of the excess mass requires that we compare the observed distribution of contracts to the counterfactual distribution that would exist in the absence of a notch. To recover the counterfactual distribution in one dimensional settings, the literature has typically fit a polynomial to the portion of the distribution that is unaffected by the discontinuity, and extrapolated
the model to the region that is impacted by the discontinuity. These predicted values constitute the counterfactual distribution. We adopt a similar approach, extending it to the joint distribution of loan size and interest rates \((r, L)\). Specifically, we observe the empirical distribution of contracts, \(F^P(r, L)\), with the goal of recovering counterfactual distribution \(F^0(r, L)\) under the Laissez-faire equilibrium.

We first take the subset of contracts from our sample that have interest rates below the lower interest rate ceiling \(\bar{r}_L\). Within this subsample, we estimate the joint distribution of \(L\) and \(\log(1 + r)\), allowing for a flexible correlation structure between the two variables. Using this joint distribution, we then predict the distribution of contracts, \((\hat{r}, \hat{L})\), for \(r > \bar{r}_L\). Note that if \(L\) and \(\log(1 + r)\) were uncorrelated, our counterfactual distribution would be equivalent to the distribution of loans for whom \(r < \bar{r}_L\).

This method relies on two central assumptions: 1) that the interest rate cap policy does not affect loan terms if the interest rate ceiling \(\bar{r}_L\) does not bind, \(F^P(r, L| r < \bar{r}_L) = F^0(r, L| r < \bar{r}_L)\), and 2) that \(F(\cdot)\) is analytic over its domain. This allows us to extrapolate the entire distribution from the conditional distribution for which \(r < \bar{r}_L\).

While the distribution of both interest rates and loan size below the lower cap appear log-normally distributed, the presence of pronounced “round number bunching” at multiples of $5,000, $10,000, etc., generates some empirical challenges. For example, in fitting a smooth lognormal counterfactual joint distribution, we will fail to reflect the “spiky” nature of \(F^P(r, L| r < \bar{r}_L)\). Therefore we take a more non-parametric approach: we divide the loan distribution into $1,000 bins, and estimate the conditional probability of a loan \(i\) being in bin \(b\) as a linear function of its interest rate, \(Pr(L_i = L^b) = \alpha^b + \beta^b r_i\). This allows us to predict the relative mass added to each bin for \(r > \bar{r}_L\).

Figure 6 compares the three relevant distributions: the full observed empirical distribution, \(F^P(r, L)\), the observed empirical distribution below the lower interest rate ceiling, \(F^P(r, L| r < \bar{r}_L)\), and the estimated counterfactual distribution, \(\hat{F}^0(r, L)\). While very similar, \(\hat{F}^0(r, L)\) corrects \(F^P(r, L| r < \bar{r}_L)\) for the correlation between loan size and interest rates – since smaller loans tend to have higher interest rates, the extrapolation over \(r > \bar{r}_L\) adds density to the lower portion of the loan distribution. Note that while these figures plot the marginal distribution of \(L\) for visual clarity, our counterfactual is defined across the joint distribution of \((L, r)\).

Figure 7 shows the difference in density between the observed loan distribution and the counterfactual distribution, pooling over all markets and loans. Here the excess mass at the threshold,
$50,000, is pronounced and equal to 2 percentage points. One can also see that there is some missing mass to the right of the threshold, where loan contracts would have been located in the absence of the discontinuity.

5.2 Estimation of Parameters

Our identification strategy relies not only on the observed excess and missing mass, but on the degree to which this excess mass varies with market concentration. Separate identification of both $\alpha$ and $\sigma$ requires that we have at least two moment equations; we use the fact that the indifference conditions are also a function of concentration, $K$.

For each market, we calculate the empirical joint probability density ($\hat{F}_k^P$) over a 2-dimensional grid, with grid points defined by the intervals $L = [0 : 1,000 : 350,000]$ and $\log(r + 1)$ where $r = [0 : 0.005 : .13]$. The visible bunching in the loan size distribution at round number multiples requires that we use this discrete, rather than continuous, approach. We calculate the counterfactual density ($\hat{F}_k^0$) over this same domain, and calculate the difference between the two as $\hat{D}_k = \hat{F}_k^P - \hat{F}_k^0$. Figure

Using $\hat{D}_k$, we calculate the empirical analogues to our theoretical indifference conditions. Specifically excess and missing mass are defined as:

$$\hat{E}_k = \sum_{i,j} \hat{D}_k(L_i, r_j) \cdot 1(r_j < \bar{r}^H)$$
\[ \hat{M}_k = \sum_{i,j} \hat{D}_k(L_i, r_j) \cdot 1 \left( \frac{(L/L_i)^{\alpha} - \bar{L}/L_i}{1+(1-1/K)\sigma} \cdot \frac{1+(1-1/K)\sigma}{1+(1-1/K)\sigma} \cdot \frac{1}{1+(1-1/K)\sigma} \cdot \frac{1}{1+(1-1/K)\sigma} \right) \]

Our estimation routine chooses \((\alpha, \sigma)\) such that:

\[
(\hat{\alpha}, \hat{\sigma}) = \arg \min R(\alpha, \sigma), \text{ where } R(\alpha, \sigma) = \sum_k (\hat{E}_k + \hat{M}_k)^2
\]

For \(K > 1\) the model is identified.

5.3 Results

Our current specification splits the data into two groups by market size, using the threshold of \(K = 4\), which provides two moment conditions to estimate our two key parameters. The preliminary estimates are shown in Table 2 – these estimates imply that in a monopolistic market there would be a 31% markup over costs, while in a market with 5 lenders the markup would be 5.5%.
6 Concluding Remarks

Access to credit is an important determinant of entrepreneurial growth, and comes at a vital point in the life-cycle of many firms. However, market power can create distortions and lead to sub-optimal provision of credit. This paper shows that, while even in the presence of significant market power intensive margin credit provision can be efficient, extensive margin responses can be inefficient. Utilizing a bunching estimator we estimate model parameters and show that excess bunching around interest rates notches is more pronounced in concentrated markets. We advance bunching approaches to environments with multi-dimensional behavioral responses, and provide general identification conditions for this approach.

Loan contracts have a number of different parameters, including not only interest rates and guarantees, but also maturity, covenants and other attributes which are affected by policy. Future work can further study multi-dimensional behavioral responses, and how lenders and borrowers respond to simultaneous changes in policy parameters.
References


A Model Simulations

Figure 8: Model Simulations of Distributional Response to an Interest Rate Ceiling, Across Markets of Varying Concentration
B SBA Express Program

The SBA Express program was established in 1995 (under the original name FA$TTRAK) and provides a 50% loan guarantee on loans up to $350k. It is the second most popular SBA lending program, besides the 7(a) guarantee program.

The primary differences between the Express program and the SBA’s flagship 7(a) Loan Program is in the maximum loan amounts, which are lower in the Express Program, the prime interest rates, which are higher in the Express program, and the SBA review time, which is typically shorter for Express loans. The documentation necessary for the SBA Express loan is less taxing compared to the standard SBA 7(a) loans, at the cost of higher interest rates.

There are two types of SBA Express loans. The first type of loans is for businesses that export goods, and the second type is for all other business. Lenders can approve a loan or line of credit up to $350,000 with an SBA Express loan. Loans can go to $500,000 if it is an Export Express Loan. The SBA Export Express loan program can help businesses that export goods get up to $500,000. The SBA will respond within 36 hours following the submission of a loan application for an Express Loan, while the eligibility review will take up to 24 hours for an Export Express Loan.

The type of loan and the type of collateral determine the amount of repayment time. The (expected) life of collateral is used to determine the repayment time: for example, using real estate for collateral is expected to lead to a longer repayment period, compared to securing a loan against equipment collateral. In particular, the maximum SBA Express loan terms are up to 25 years for real estate term loans, up to 10 years for leasehold improvement term loans, ranging between 10 and 25 years for equipment, fixtures or furniture term loans, up to 10 years for inventory or working capital term loans, and up to 7 years for revolving lines of credit.